

Exercise walk-through

6.4

Sampling and estimation

A-Level Mathematics

Continuous random variables

A density function

$$f(x) \geq 0, \int f = 1.$$

$$P(a < X < b) = \int_a^b f, \quad E(X) = \int xf.$$

Median

solves $F(m) = 0.5$.

The two functions of a continuous variable

Probability density function

The **probability density function** *area* beneath it, and the total area is 1.

$f(x)$: probability is the

Cumulative distribution function

The **cumulative distribution function** – the integral of f , and differentiating F gives f back.

$F(x) = P(X \leq x)$

Expectation

The **expectation** non-zero.

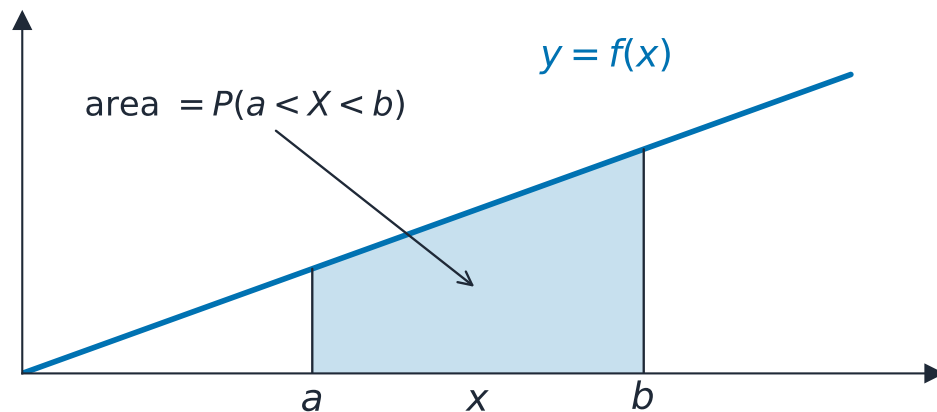
is $E(X) = \int xf(x) dx$ over the range where f is

The normal distribution

The **normal distribution** is the case whose density is that symmetric bell – and the one every table is written for.

Medians and **percentiles** come from F , means and variances from f . Knowing which one a question needs saves most of the work.

Continuous random variables, in detail



$$\int_{-\infty}^{\infty} f = 1 \quad .$$

$$P(a < X < b) = \int_a^b f$$

A **density function** is never negative and encloses **unit area**: $f(x) \geq 0$ and $\int f = 1$. Probability is an **area**, so $P(X = a) = 0$ for any single value.

Sampling and estimation

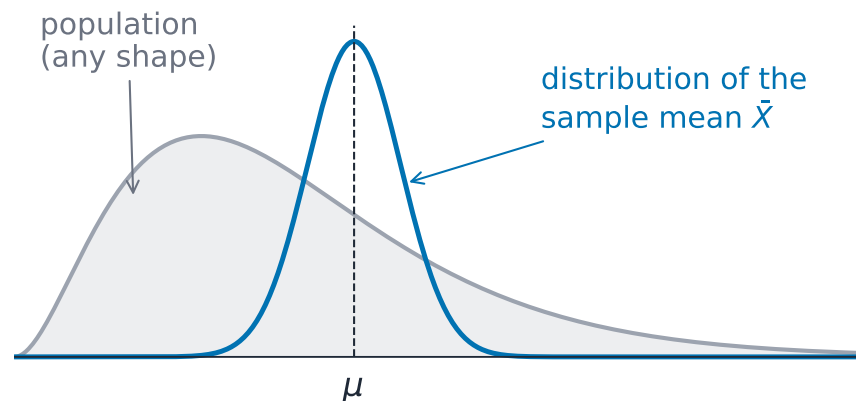
$E(\bar{X}) = \mu$, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$. A 95% **confidence interval** :

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}.$$

$$n = 64, \bar{x} = 50, \sigma = 8$$

$$50 \pm 1.96 \Rightarrow (48.0, 52.0)$$

Sampling and estimation, in detail



CLT: $\bar{X} \approx N(\mu, \sigma^2/n)$ for large n · $SE = \sigma/\sqrt{n}$

CLT : $\bar{X}$ near-normal

$$E(\bar{X}) = \mu, \text{Var}(\bar{X}) = \frac{\sigma^2}{n}.$$

Exercises 2.1·2.2

2.1 A population has $\sigma = 10$. For a sample of $n = 25$, find the standard error $\frac{\sigma}{\sqrt{n}}$. [2]

2.2 State the value of z used in a 95% confidence interval. [1]

Exercises 2.1·2.2

2.1 A population has $\sigma = 10$. For a sample of $n = 25$, find the standard error $\frac{\sigma}{\sqrt{n}}$. [2]

2.1

- $\frac{10}{\sqrt{25}} = \frac{10}{5} = 2$

2.2 State the value of z used in a 95% confidence interval. [1]

2.2

- 1.96

Exercise 3.1

3.1 As the sample size n increases, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$:

[1]

☐

A increases

B decreases

C stays the same

D becomes negative

Exercise 3.1

3.1 As the sample size n increases, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$: [1]

☐

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3.1 B

- Dividing by a larger n makes $\text{Var}(\bar{X})$ smaller

Exercise 3.2 (a)

3.2 A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(a) Find a 95% confidence interval for the mean lifetime.

[3]

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(a) Find a 95% confidence interval for the mean lifetime. [3]

(a)

- $1200 \pm 1.96 \frac{150}{\sqrt{100}} = 1200 \pm 1.96(15) = 1200 \pm 29.4; (1170.6, 1229.4)$

Exercise 3.2 (b)

3.2 A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(b) State what changes to the interval if a 99% level ($z = 2.576$) is used instead. [2]

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3.2 A random sample of 100 light bulbs has mean lifetime $\bar{x} = 1200$ hours. The population standard deviation is $\sigma = 150$ hours.

(b) State what changes to the interval if a 99% level ($z = 2.576$) is used instead. [2]

(b)

- the interval becomes **wider** (a higher confidence level needs a larger z , here 2.576)

Exercise 3.3

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- mean $\bar{x} = \frac{120}{8} = 15$; unbiased variance $s^2 = \frac{1}{n-1} \left(\sum x^2 - \frac{(\sum x)^2}{n} \right) = \frac{1}{7} \left(1840 - \frac{120^2}{8} \right)$
 $\frac{1}{7}(1840 - 1800) = \frac{40}{7} = 5.71$