

Exercise walk-through

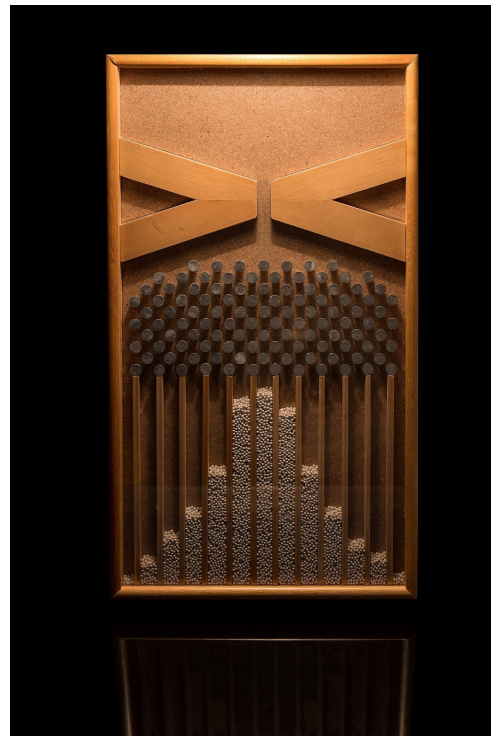
5.4

Discrete random variables

A-Level Mathematics

What we will cover

- 1 Representing data
- 2 Permutations & combinations
- 3 Probability
- 4 Discrete random variables
- 5 The normal distribution
- 6 Recap



The named discrete distributions

Probability distribution table

Any discrete random variable can be written as a **probability distribution table** : every value, with its probability, summing to 1.

Expectation

The **expectation** $E(X) = \sum xP(X = x)$ – the long-run mean, not a value the variable need ever take.

Binomial distribution

The **binomial distribution** $B(n, p)$: a fixed number of independent trials, each a success or failure with the same p .

Geometric distribution

The **geometric distribution** counts trials *up to and including* the first success, so $P(X = r) = p(1 - p)^{r-1}$.

Choose by the question asked: how many successes in n trials, or how many trials until the first success.

Exam tips

- Decide whether order matters: **permutations** (${}^nP_r$) **when it does**, **combinations** (${}^nC_r$) when it does not.
- For a **discrete random variable**, check the probabilities sum to 1 and use $E(X) = \sum xP(X=x)$.
- For the **normal distribution**, standardise with $z = (x - \mu)/\sigma$, **sketch and shade**, then read the table.
- Apply a **continuity correction** when approximating a discrete variable by the normal.

Exercises 2.1·2.2

2.1 $X \sim B(8, 0.25)$. Find $E(X)$. [1]

2.2 A geometric variable has $p = 0.2$. Find $E(X)$. [1]

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2.1

- $E(X) = np = 8 \times 0.25 = 2$

2.2 A geometric variable has $p = 0.2$. Find $E(X)$. [1]

2.2

- $E(X) = \frac{1}{p} = \frac{1}{0.2} = 5$

Exercise 3.1

3.1 For $X \sim B(12, 0.5)$, the variance $\text{Var}(X)$ is:

[1]

☐

A 6

B 3

C 12

D 1.5

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3.1 B

- $\text{Var}(X) = np(1 - p) = 12(0.5)(0.5) = 3$

Exercise 3.2 (a)

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(a) Find k .

[1]

Exercise 3.2 (a)

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(a) Find k .

[1]

(a)

- probabilities sum to 1: $0.1 + 0.3 + k + 0.2 = 1 \Rightarrow k = 0.4$

Exercise 3.2 (b)

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(b) Find $E(X)$ and $\text{Var}(X)$.

[5]

Exercise 3.2 (b)

3.2 A discrete random variable X has the distribution:

x	1	2	3	4
$P(X = x)$	0.1	0.3	k	0.2

(b) Find $E(X)$ and $\text{Var}(X)$.

[5]

(b)

- $E(X) = 1(0.1) + 2(0.3) + 3(0.4) + 4(0.2) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7$; $E(X^2) = 1(0.1) + 4(0.3) + 9(0.4) + 16(0.2) = 0.1 + 1.2 + 3.6 + 3.2 = 8.1$; $\text{Var}(X) = 8.1 - 2.7^2 = 8.1 - 7.29 = 0.81$

Exercise 3.3

3.3 A fair coin is tossed 6 times. Find the probability of obtaining exactly 4 heads. [3]

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3.3

- $X \sim B(6, 0.5); P(X = 4) = \binom{6}{4}(0.5)^6 = 15 \times \frac{1}{64} = \frac{15}{64} = 0.234$