Exercise walk-through

5.3 Probability

A-Level Mathematics

Permutations and combinations

Permutation (order matters): ${}^nP_r = \frac{n!}{(n-r)!}$.

Combination : ${}^nC_r = \binom{n}{r}$.

NEEDLESS (E×3, S×2)

$$\frac{8!}{3!\,2!} = 3360$$

permutation (order matters)

AB BA

combination (order does not)

NPr = n!/(n−r)!; nCr = nPr/r!

Arranging 3 of 4 objects ($^4P_3$)

$$\boxed{4} \times \boxed{3} \times \boxed{2} = 24$$

1st pick 2nd pick 3rd pick

Choosing 3 of 4 (order ignored): $^4C_3 = \frac{^4P_3}{3!} = 4$

$$^nP_r = n!/(n-r)! \quad \cdot \quad ^nC_r = {}^nP_r/r!$$

nPr = n!/(n−r)!; nCr = nPr/r!

Worked example – arrangements with repeated letters

How many different arrangements are there of the letters of NEEDLESS?

- 8 letters, with E three times and S twice
- divide by the factorial of each repeat count
- $\dfrac{8!}{3!\,2!} = \dfrac{40320}{12} = 3360$

Repeated letters are indistinguishable, so every arrangement has been counted $3! \times 2!$ times. Dividing by the repeats is the whole correction.

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

Independent: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$. Conditional: $\mathrm{P}(A \mid B) = \mathrm{P}(A \cap B)/\mathrm{P}(B)$.

> Mutually exclusive $\Rightarrow$ intersection is empty, so you just add. Independent is not the same as exclusive.

Exercises 2.1·2.2

2.1 Events are mutually exclusive with $P(A) = 0.3$, $P(B) = 0.45$. Find $P(A \cup B)$. [2]

2.2 State the test for two events A and B to be independent. [1]

Exercises 2.1·2.2

2.1 Events are mutually exclusive with $P(A) = 0.3$, $P(B) = 0.45$. Find $P(A \cup B)$. [2]

2.1

- mutually exclusive
- $P(A \cup B) = 0.3 + 0.45 = 0.75$

2.2 State the test for two events A and B to be independent. [1]

2.2

- $P(A \cap B) = P(A) \times P(B)$

Exercise 3.1

3.1 For independent events with $P(A) = 0.6$ and $P(B) = 0.5$, $P(A \cap B)$ is: [1]

- [ ] **A** 1.1
- [ ] **B** 0.3
- [ ] **C** 0.1
- [ ] **D** 0

Exercise 3.1

3.1 For independent events with $P(A) = 0.6$ and $P(B) = 0.5$, $P(A \cap B)$ is: [1]

- [ ] **A** 1.1
- [ ] **B** 0.3
- [ ] **C** 0.1
- [ ] **D** 0

3.1 B

- $0.6 \times 0.5 = 0.3$

Exercise 3.2 (a)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$. [2]

Exercise 3.2 (a)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$. [2]

(a)

- $0.8 = 0.6 + 0.5 - P(A \cap B) \Rightarrow P(A \cap B) = 0.3$

Exercise 3.2 (b)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(b) Determine whether A and B are independent. [2]

Exercise 3.2 (b)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(b) Determine whether A and B are independent. [2]

(b)

- $P(A)P(B) = 0.6 \times 0.5 = 0.3 = P(A \cap B)$, so they **are** independent

Exercise 3.2 (c)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(c) Find $P(A \mid B)$. [2]

Exercise 3.2 (c)

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(c) Find $P(A \mid B)$. [2]

(c)

- $P(A \mid B) = \dfrac{0.3}{0.5} = 0.6$

Exercise 3.3

3.3 A bag has 5 red and 3 blue balls. Two are drawn without replacement. Find the probability that both are red. [3]

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3.3

- $P(\text{both red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$