

Exercise walk-through

5.2

Permutations and combinations

A-Level Mathematics

Permutations and combinations

Permutation (order matters): ${}^nP_r = \frac{n!}{(n-r)!}$.

Combination : ${}^nC_r = \binom{n}{r}$.

NEEDLESS (E×3, S×2)

$$\frac{8!}{3!2!} = 3360$$

permutation (order matters)

AB

BA

combination (order does not)

$$nPr = n!/(n-r)!; nCr = nPr/r!$$

Arranging 3 of 4 objects (${}^4P_3$)

$$\boxed{4} \times \boxed{3} \times \boxed{2} = 24$$

1st pick 2nd pick 3rd pick

Choosing 3 of 4 (order ignored): ${}^4C_3 = \frac{{}^4P_3}{3!} = 4$

$${}^nP_r = n!/(n-r)! \quad \cdot \quad {}^nC_r = {}^nP_r/r!$$

$$nPr = n!/(n-r)!; nCr = nPr/r!$$

Worked example – arrangements with repeated letters

How many different arrangements are there of the letters of NEEDLESS?

- 8 letters, with E three times and S twice
- divide by the factorial of each repeat count

- $\frac{8!}{3! 2!} = \frac{40320}{12} = 3360$

Repeated letters are indistinguishable, so every arrangement has been counted $3! \times 2!$ times. **Dividing by the repeats** is the whole correction.

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Independent: $P(A \cap B) = P(A)P(B)$. Conditional: $P(A | B) = P(A \cap B)/P(B)$.

Mutually exclusive $\Rightarrow$ intersection is empty, so you just add. Independent is not the same as exclusive.

Exercises 2.1·2.2

2.1 Evaluate ${}^7P_3$.

[1]

2.2 Evaluate ${}^8C_2$.

[1]

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2.1 Evaluate ${}^7P_3$.

[1]

2.1

- ${}^7P_3 = 7 \times 6 \times 5 = 210$

2.2 Evaluate ${}^8C_2$.

[1]

2.2

- ${}^8C_2 = \frac{8 \times 7}{2} = 28$

Exercise 3.1

3.1 The number of ways to arrange the letters of BANANA is:

[1]

☐

A 720

B 120

C 60

D 360

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[1]

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3.1 C

- BANANA has 6 letters with $A \times 3$, $N \times 2$: $\frac{6!}{3! 2!} = \frac{720}{12} = 60$

Exercise 3.2 (a)

3.2 A committee of 4 is chosen from 7 women and 5 men.

(a) Find the number of possible committees.

[2]

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[2]

(a)

- ${}^{12}C_4 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{24} = 495$

Exercise 3.2 (b)

3.2 A committee of 4 is chosen from 7 women and 5 men.

(b) Find the number that contain exactly 2 women and 2 men.

[3]

Exercise 3.2 (b)

3.2 A committee of 4 is chosen from 7 women and 5 men.

(b) Find the number that contain exactly 2 women and 2 men.

[3]

(b)

- ${}^7C_2 \times {}^5C_2 = 21 \times 10 = 210$

Exercise 3.3

3.3 Find the number of arrangements of the 7 letters of PICTURE in which the three vowels (I, U, E) are all together. [3]

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3.3

- treat (IUE) as one block
- 5 items arrange in $5! = 120$ ways; the vowels arrange among themselves in $3! = 6$ ways; total $= 120 \times 6 = 720$