

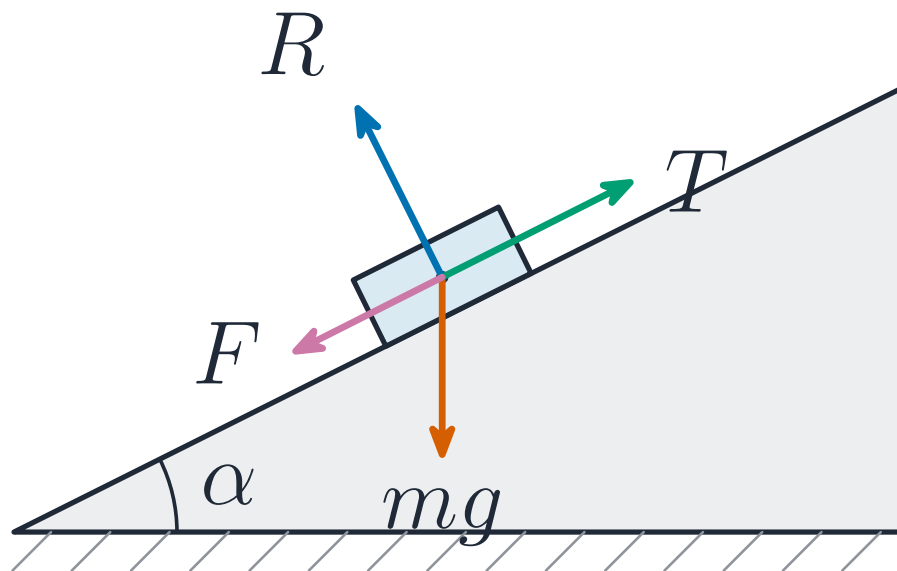
Exercise walk-through

4.5

Energy, work and power

A-Level Mathematics

Newton's laws



$$F = ma, \quad W = mg.$$

On an **inclined plane** :
resolve along and perpendicular to
the slope, then apply $F = ma$.

**Pull 12 kg up a 20° rough
slope**

$$T \approx 110 \text{ N}$$

The laws, and the forces they name

Newton's laws of motion

Newton's laws of motion : a body keeps its velocity unless a **resultant** force acts; $F = ma$; and every force has an equal and opposite partner.

Newton's third law

Newton's third law pairs act on *different* bodies – which is why they never cancel in one free-body diagram.

Connected particles

For **connected particles** write $F = ma$ for each body separately, with the same acceleration and the tension appearing twice, in opposite directions.

Thrust

A light rod can push as well as pull: the compression force is **thrust**, and a string can never provide it.

Limiting equilibrium is the instant before sliding: friction is at its maximum μR , and the body is still in equilibrium.

Energy, work and power

$$W = Fd \cos \theta, \quad \text{KE} = \frac{1}{2}mv^2, \quad \text{PE} = mgh.$$

No friction $\Rightarrow$ total energy conserved. Power
 $P = Fv$.

12 kW engine at 20 m/s

$$F = \frac{P}{v} = 600 \text{ N}$$

The energy quantities you can calculate

kinetic energy

$$\frac{1}{2}mv^2$$

gravitational potential energy

 mgh , measured from a stated level

work done

 Fd – force times distance **along the force**

power

$$\frac{\text{work}}{\text{time}} = Fv \text{ for a constant force}$$

conservation of energy

with no resistance, $KE + PE$ is constant

with resistance

work done against **air resistance**
the energy lost from that total

or friction is

On a **velocity-time graph** the **area** is the displacement and the **gradient** is the acceleration, with a negative gradient meaning **deceleration**.

Forces and equilibrium, continued



Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

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Exercises 2.1·2.2

2.1 Find the kinetic energy of a 2 kg body moving at 6 m s^{-1} . [2]

2.2 A force of 50 N moves a body 4 m in its own direction. Find the work done. [1]

Exercises 2.1·2.2

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2.1

- $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(2)(36) = 36 \text{ J}$

2.2 A force of 50 N moves a body 4 m in its own direction. Find the work done. [1]

2.2

- $W = Fd = 50 \times 4 = 200 \text{ J}$

Exercise 3.1

3.1 A 1500 W motor raises a load at constant speed. In 10 s it does work of: [1]

☐

A 150 J

B 1500 J

C 15000 J

D 150000 J

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3.1 C

- $\text{work} = \text{power} \times \text{time} = 1500 \times 10 = 15000 \text{ J}$

Exercise 3.2 (a)

3.2 A car of mass 1200 kg accelerates from 10 m s^{-1} to 20 m s^{-1} on a level road.

(a) Find the increase in kinetic energy.

[3]

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(a)

- $\Delta \text{KE} = \frac{1}{2}(1200)(20^2 - 10^2) = 600(400 - 100) = 600(300) = 180000 \text{ J}$

Exercise 3.2 (b)

3.2 A car of mass 1200 kg accelerates from 10 m s^{-1} to 20 m s^{-1} on a level road.

(b) If a constant resistance of 400 N acts over the 150 m taken, find the total work done by the engine. [3]

Exercise 3.2 (b)

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(b)

- work by engine = ΔKE + work against resistance = $180000 + 400(150)$; = $180000 + 60000 = 240000 \text{ J}$

Exercise 3.3

3.3 A cyclist and bicycle of total mass 90 kg freewheel from rest down a slope, dropping a vertical height of 20 m. Resistances do 9000 J of work.

(a) Using energy, find the speed at the bottom.

[4]

Exercise 3.3

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3.3

(a)

- energy: $mgh - (\text{resistance work}) = \frac{1}{2}mv^2$; $90(10)(20) - 9000 = \frac{1}{2}(90)v^2$; $18000 - 9000 = 45v^2 \Rightarrow v^2 = 200$; $v = 14.1 \text{ m s}^{-1}$