

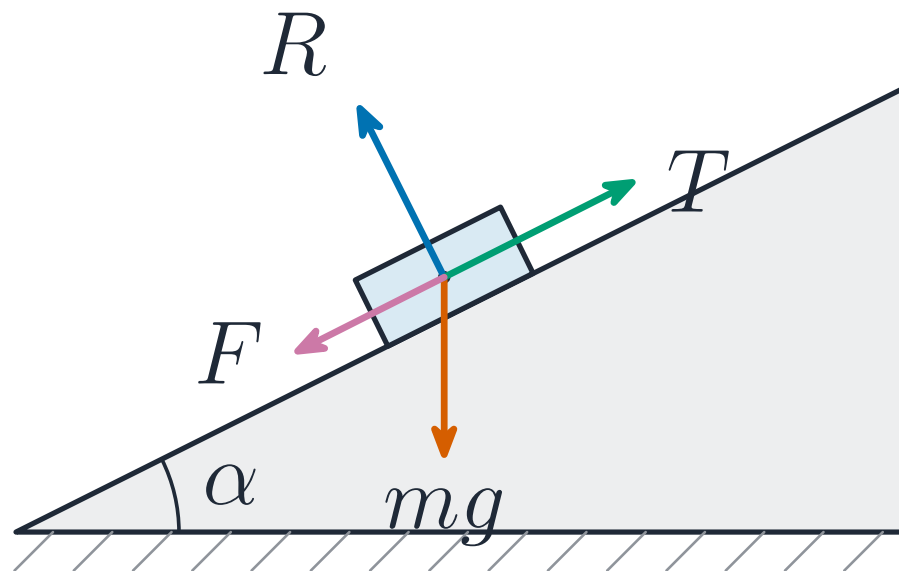
Exercise walk-through

4.1

Forces and equilibrium

A-Level Mathematics

Newton's laws



$$F = ma, \quad W = mg.$$

On an **inclined plane** :
resolve along and perpendicular to
the slope, then apply $F = ma$.

**Pull 12 kg up a 20° rough
slope**

$$T \approx 110 \text{ N}$$

The laws, and the forces they name

Newton's laws of motion

Newton's laws of motion : a body keeps its velocity unless a **resultant** force acts; $F = ma$; and every force has an equal and opposite partner.

Newton's third law

Newton's third law pairs act on *different* bodies – which is why they never cancel in one free-body diagram.

Connected particles

For **connected particles** write $F = ma$ for each body separately, with the same acceleration and the tension appearing twice, in opposite directions.

Thrust

A light rod can push as well as pull: the compression force is **thrust**, and a string can never provide it.

Limiting equilibrium is the instant before sliding: friction is at its maximum μR , and the body is still in equilibrium.

Energy, work and power

$$W = Fd \cos \theta, \quad \text{KE} = \frac{1}{2}mv^2, \quad \text{PE} = mgh.$$

No friction $\Rightarrow$ total energy conserved. Power
 $P = Fv$.

12 kW engine at 20 m/s

$$F = \frac{P}{v} = 600 \text{ N}$$

The energy quantities you can calculate

kinetic energy

$$\frac{1}{2}mv^2$$

gravitational potential energy

 mgh , measured from a stated level

work done

 Fd – force times distance **along the force**

power

$$\frac{\text{work}}{\text{time}} = Fv \text{ for a constant force}$$

conservation of energy

with no resistance, $KE + PE$ is constant

with resistance

work done against **air resistance**
the energy lost from that total

or friction is

On a **velocity-time graph** the **area** is the displacement and the **gradient** is the acceleration, with a negative gradient meaning **deceleration**.

Forces and equilibrium, continued



Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

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Exercises 2.1-2.3

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

2.3 A force of 20 N acts at 35° above the horizontal. Find its horizontal and vertical components. [2]

Exercises 2.1·2.3

2.1 A force of 20 N acts at 60° above the horizontal. Find its horizontal component. [2]

2.1

- $20 \cos 60^\circ = 20 \times 0.5 = 10 \text{ N}$

2.2 A block of weight 40 N rests on a rough surface, $\mu = 0.25$. Find the maximum friction available. [2]

2.2

- $F_{\max} = \mu R = 0.25 \times 40 = 10 \text{ N}$

2.3 A force of 20 N acts at 35° above the horizontal. Find its horizontal and vertical components. [2]

2.3

- Horizontal = $20 \cos 35^\circ = 16.4 \text{ N}$; vertical = $20 \sin 35^\circ = 11.5 \text{ N}$

Exercises 2.4·2.5

2.4 A block of mass 5 kg rests on a rough horizontal table with $\mu = 0.4$. Find the least horizontal force that would move it. [3]

2.5 State the condition under which $F = \mu R$ rather than $F \leq \mu R$. [1]

Exercises 2.4·2.5

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2.4

- $R = 5(9.81) = 49.05 \text{ N}$; $F = \mu R = 0.4(49.05) = 19.6 \text{ N}$
-

2.5 State the condition under which $F = \mu R$ rather than $F \leq \mu R$. [1]

2.5

- When the object is in limiting equilibrium – on the point of sliding

Exercise 3.1

3.1 A particle is in equilibrium under three forces. The sum of the forces is: [1]

☐

A maximum

B zero

C equal to the weight

D equal to μR

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3.1 B

- In equilibrium the resultant (vector sum) of the forces is zero

Exercise 3.2 (a)

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(a) Find the normal reaction.

[1]

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(a)

- $R = mg = 8 \times 10 = 80 \text{ N}$

Exercise 3.2 (b)

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

Exercise 3.2 (b)

3.2 A box of mass 8 kg rests on a rough horizontal floor. A horizontal force of 30 N is applied but the box does not move.

(b) Given the box is on the point of slipping, find the coefficient of friction. [3]

(b)

- on the point of slipping $F = \mu R$ and $F = 30$ N; $\mu = \frac{30}{80} = 0.375$

Exercise 3.3

3.3 A particle of weight W hangs in equilibrium from two light strings making angles 30° and 60° with the horizontal. The tensions are T_1 and T_2 . Given $W = 20$ N, find T_1 and T_2 . [5]

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3.3

- resolve: horizontal $T_2 \cos 60^\circ = T_1 \cos 30^\circ$; vertical $T_1 \sin 30^\circ + T_2 \sin 60^\circ = 20$; from the first $T_2(0.5) = T_1(0.866) \Rightarrow T_2 = 1.732 T_1$; substitute: $0.5T_1 + 0.866(1.732T_1) = 20 \Rightarrow 2T_1 = 20 \Rightarrow T_1 = 10$ N, $T_2 = 17.3$ N

Exercise 3.4 (a)

3.4 A particle of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.35. A force P acts on the particle **up the line of greatest slope**. Take $g = 9.81 \text{ m s}^{-2}$.

(a) Find the normal contact force.

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[2]

(a)

- Resolving perpendicular to the plane: $R = 6(9.81) \cos 25^\circ = 53.3 \text{ N}$

Exercise 3.4 (b)

3.4 A particle of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.35. A force P acts on the particle **up the line of greatest slope**. Take $g = 9.81 \text{ m s}^{-2}$.

(b) Find the value of P for which the particle is on the point of sliding **down** the plane.^[4]

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(b) Find the value of P for which the particle is on the point of sliding **down** the plane.^[4]

(b)

- On the point of sliding down, friction acts up the plane at its maximum: $F = \mu R = 0.35(53.3) = 18.7 \text{ N}$; resolving along the plane, $P + F = mg \sin 25^\circ$; $mg \sin 25^\circ = 6(9.81) \sin 25^\circ = 24.9 \text{ N}$; $P = 24.9 - 18.7 = 6.2 \text{ N}$

Exercise 3.4 (c)

3.4 A particle of mass 6 kg rests on a rough plane inclined at 25° to the horizontal. The coefficient of friction between the particle and the plane is 0.35. A force P acts on the particle **up the line of greatest slope**. Take $g = 9.81 \text{ m s}^{-2}$.

(c) Find the value of P for which the particle is on the point of sliding **up** the plane. [3]

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(c) Find the value of P for which the particle is on the point of sliding **up** the plane. [3]

(c)

- On the point of sliding up, friction acts down the plane: $P = mg \sin 25^\circ + F = 24.9 + 18.7 = 43.6 \text{ N}$

Exercise 3.4 (d)

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(d) Hence state the range of values of P for which the particle remains in equilibrium.[1]

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(d)

- $6.2 \text{ N} \leq P \leq 43.6 \text{ N}$

Exercise 3.4 (e)

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(e) Explain why the friction force changes direction between parts (b) and (c). [2]

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(e)

- Friction always opposes the motion that would occur if it were absent; with a small P the particle would slide down so friction acts up the plane, and with a large P it would slide up so friction acts down