

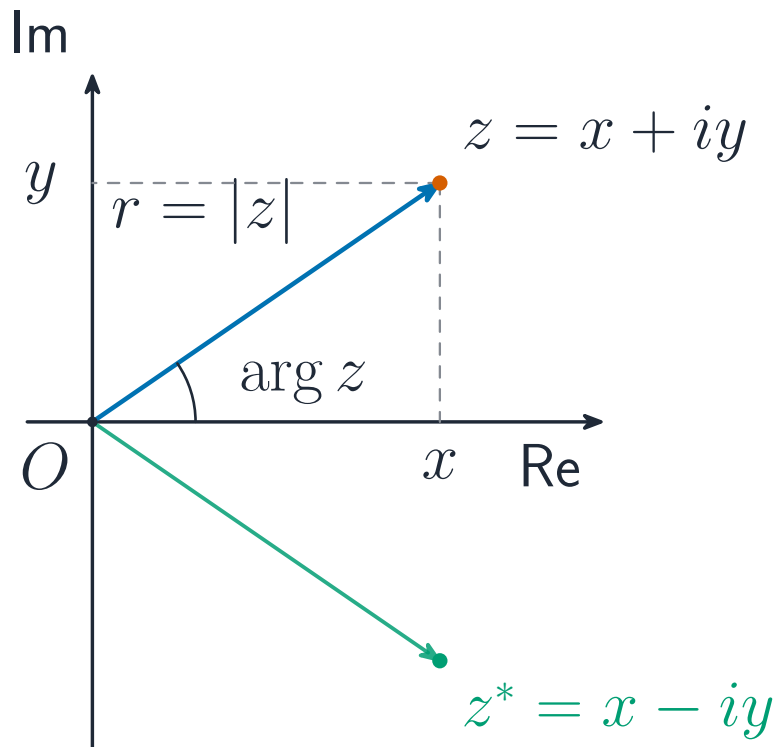
Exercise walk-through

3.9

Complex numbers

A-Level Mathematics

Complex numbers



$z = x + iy$, $i^2 = -1$. **Conjugate**
 $z^* = x - iy$.

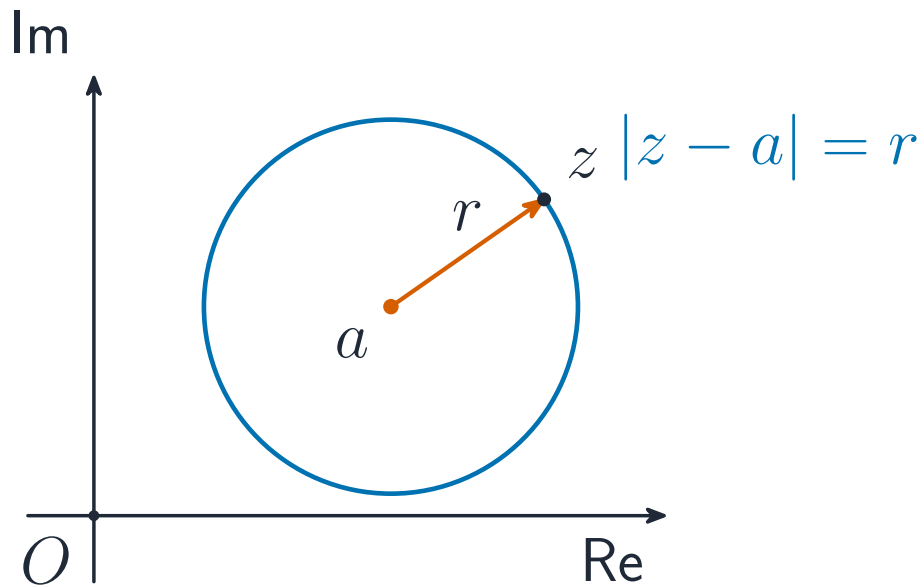
$$|z| = \sqrt{x^2 + y^2}, \quad z = re^{i\theta}.$$

Divide by multiplying by the conjugate:

$$\frac{3 + i}{1 - i}$$

$$= 1 + 2i$$

Loci on the Argand diagram



A condition on z draws a **locus** :

- $|z - a| = r$ – a circle
- $|z - a| = |z - b|$ – a perpendicular bisector
- $\arg(z - a) = \theta$ – a half-line

Complex numbers: the five things to know

Cartesian form

$z = x + iy$; x is the real part, y the **imaginary part**

the conjugate

$z^* = x - iy$. For a polynomial with **real** coefficients, non-real roots come in **conjugate pairs**

the modulus

$|z| = \sqrt{x^2 + y^2}$ – its distance from the origin

the argument

the angle the point makes, measured from the positive real axis

polar form

$z = r(\cos \theta + i \sin \theta) = re^{i\theta}$ – multiplying multiplies the moduli and **adds** the arguments

on the Argand diagram

$|z|$ is the distance from O , $\arg z$ the angle, and z^* the **reflection in the real axis**

Choose the form by the operation: **Cartesian** for adding, **polar** for multiplying, dividing and taking powers.

Worked example – integration by parts

Find $\int x \cos x \, dx$.

By parts with $u = x$, $\frac{dv}{dx} = \cos x$ (so $v = \sin x$):

$$\begin{aligned}\int x \cos x \, dx &= x \sin x - \int \sin x \, dx \\ &= x \sin x + \cos x + C\end{aligned}$$

Choose u to be the part that gets **simpler** when differentiated – here $x \rightarrow 1$. The other part must be integrable.

Partial fractions

$$\frac{x+4}{(x+1)(x-2)} \xrightarrow{\text{split}} \frac{2}{x-2} - \frac{1}{x+1}$$

One fraction with a factorised bottom splits into a sum of simpler fractions.

Exercises 2.1·2.2

2.1 Find the modulus of $z = 3 + 4i$. [1]

2.2 Write $(2 + i)(3 - i)$ in the form $x + iy$. [2]

Exercises 2.1·2.2

2.1 Find the modulus of $z = 3 + 4i$. [1]

2.1

- $|z| = \sqrt{3^2 + 4^2} = 5$

2.2 Write $(2 + i)(3 - i)$ in the form $x + iy$. [2]

2.2

- $(2 + i)(3 - i) = 6 - 2i + 3i - i^2 = 7 + i$

Exercise 3.1

3.1 The argument of $z = -1 + i$ (in radians, $-\pi < \arg z \leq \pi$) is:

[1]

☐

A $\frac{\pi}{4}$

B $\frac{3\pi}{4}$

C $-\frac{\pi}{4}$

D $\frac{\pi}{2}$

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3.1 B

- The point $(-1, 1)$ is in the second quadrant: $\arg z = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Exercise 3.2 (a)

3.2 The complex number z satisfies $z = \frac{10}{2 - i}$.

(a) Express z in the form $x + iy$.

[3]

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(a) Express z in the form $x + iy$.

[3]

(a)

- $$z = \frac{10(2 + i)}{(2 - i)(2 + i)} = \frac{20 + 10i}{5} = 4 + 2i$$

Exercise 3.2 (b)

3.2 The complex number z satisfies $z = \frac{10}{2 - i}$.

(b) Find $|z|$ and $\arg z$ (in radians).

[3]

Exercise 3.2 (b)

3.2 The complex number z satisfies $z = \frac{10}{2 - i}$.

(b) Find $|z|$ and $\arg z$ (in radians).

[3]

(b)

- $|z| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}; \arg z = \tan^{-1} \frac{2}{4} = 0.4636 \text{ rad}$

Exercise 3.3 (a)

3.3 On a single Argand diagram, sketch the locus of points z for which:

(a) $|z - (2 + i)| = 2$.

[2]

Exercise 3.3 (a)

3.3 On a single Argand diagram, sketch the locus of points z for which:

(a) $|z - (2 + i)| = 2$.

[2]

(a)

- a circle of radius 2 centred at the point $(2, 1)$

Exercise 3.3 (b)

3.3 On a single Argand diagram, sketch the locus of points z for which:

(b) Describe the locus $|z - 3| = |z + 1|$.

[2]

Exercise 3.3 (b)

3.3 On a single Argand diagram, sketch the locus of points z for which:

(b) Describe the locus $|z - 3| = |z + 1|$. [2]

(b)

- the perpendicular bisector of the segment joining 3 and -1 – i.e. the vertical line $x = 1$