

Exercise walk-through

3.7

Vectors

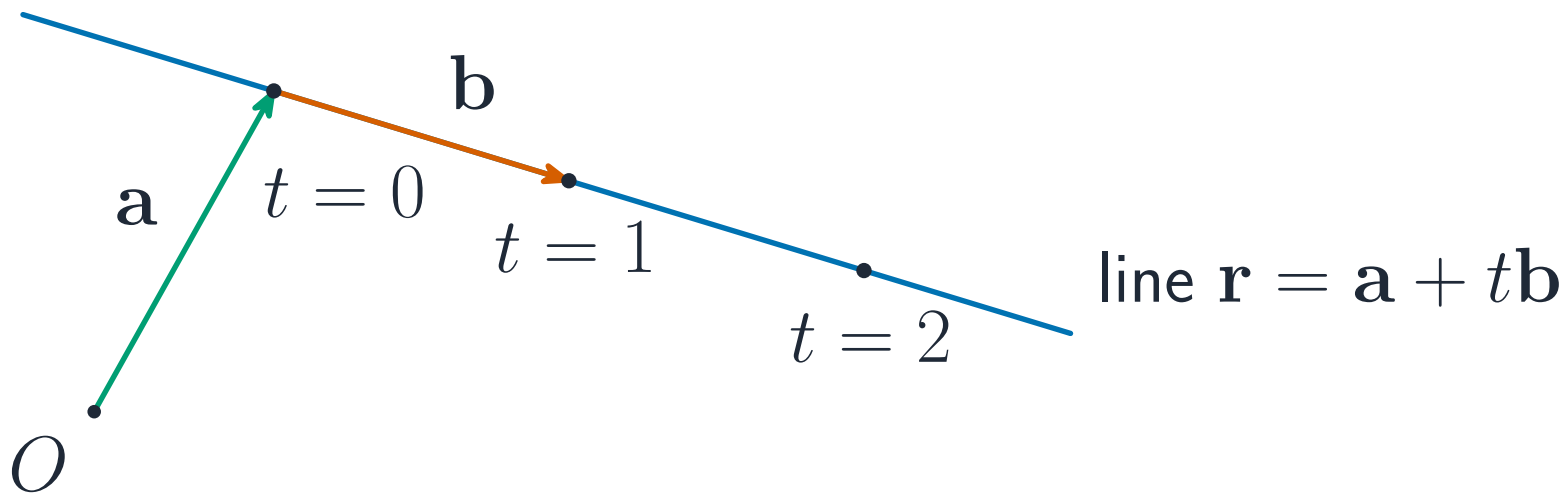
A-Level Mathematics

Vectors

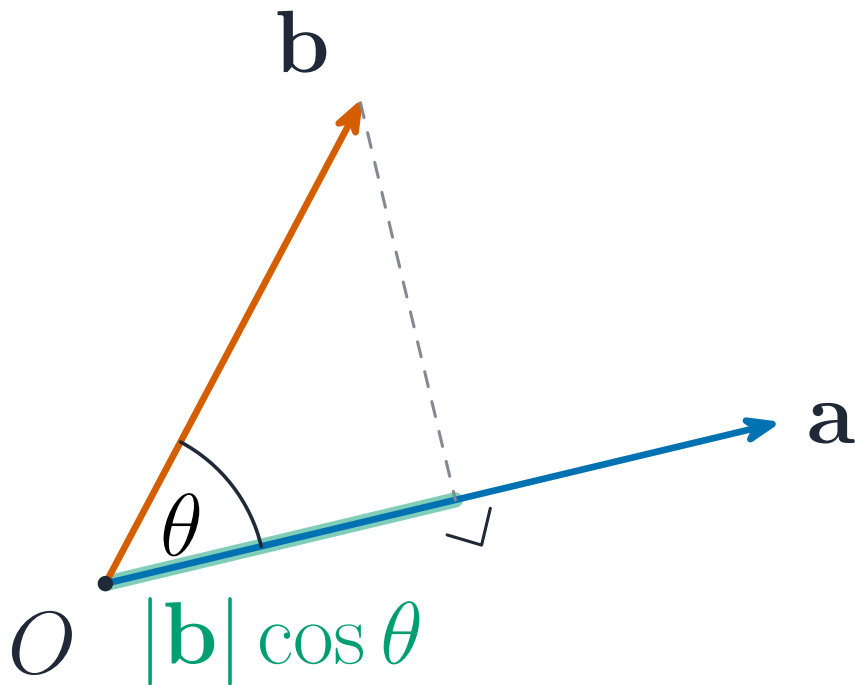
$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$; a **unit vector** has length 1.

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \text{ (a line).}$$

Lines can be parallel, intersect, or be **skew**.



The scalar product



$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta.$$

$\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow$ perpendicular.

e.g. angle between $(1, 2, 2)$ and $(2, 2, 1)$: $\cos \theta = \frac{8}{9}$, $\theta = 27.3^\circ$.

Exercises 2.1·2.2

2.1 Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$.

[2]

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

[2]

Exercises 2.1·2.2

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[2]

2.1

- $|\mathbf{v}| = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

[2]

2.2

- $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 1 = 0$

Exercise 3.1

3.1 Two vectors are perpendicular when their scalar product is:

[1]

☐

A 1

B 0

C the product of their magnitudes

D undefined

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A 1

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3.1 B

- Perpendicular vectors have scalar product 0

Exercise 3.2 (a)

3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = -3$. [2]

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(a) Find the position vector of the point on l where $t = -3$. [2]

/ 2 \

(a)

$$\bullet \quad \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ 9 \end{pmatrix}$$

Exercise 3.2 (b)

3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. [4]

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(b)

- direction of l is $\mathbf{b} = (1, 2, -2)$; $\mathbf{b} \cdot \mathbf{d} = (1)(2) + (2)(-1) + (-2)(2) = 2 - 2 - 4 = -4$;
 $|\mathbf{b}| = \sqrt{1 + 4 + 4} = 3$ and $|\mathbf{d}| = \sqrt{4 + 1 + 4} = 3$; $\cos \theta = \frac{-4}{9}$, so $\theta = 116.4^\circ$ – obtuse. The **acute** angle between the two directions is therefore $180^\circ - 116.4^\circ = 63.6^\circ$; equivalently, take the modulus first: $\cos \theta = \frac{4}{9}$

Exercise 3.3 (a)

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

[2]

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[2]

(a)

- $\overrightarrow{AB} = (2, 1, 3)$, $\overrightarrow{AC} = (1, -2, 2)$

Exercise 3.3 (b)

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(b) Find angle BAC .

[3]

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[3]

(b)

- $\vec{AB} \cdot \vec{AC} = 2 - 2 + 6 = 6; |\vec{AB}| = \sqrt{14}, |\vec{AC}| = 3; \cos(BAC) = \frac{6}{3\sqrt{14}} = 0.5345 \Rightarrow$
 $BAC = 57.7^\circ$