

Exercise walk-through

3.4

Differentiation

A-Level Mathematics

Partial fractions: check the degree first

the precondition

the top must be of **lower degree** than the bottom

if it is not

“top-heavy”: **divide** the polynomial first, giving a **quotient** plus a proper remainder

distinct linear factors

$$\frac{A}{ax + b} + \frac{B}{cx + d}$$

a repeated factor

$$\frac{A}{ax + b} + \frac{B}{(ax + b)^2}$$

an irreducible quadratic

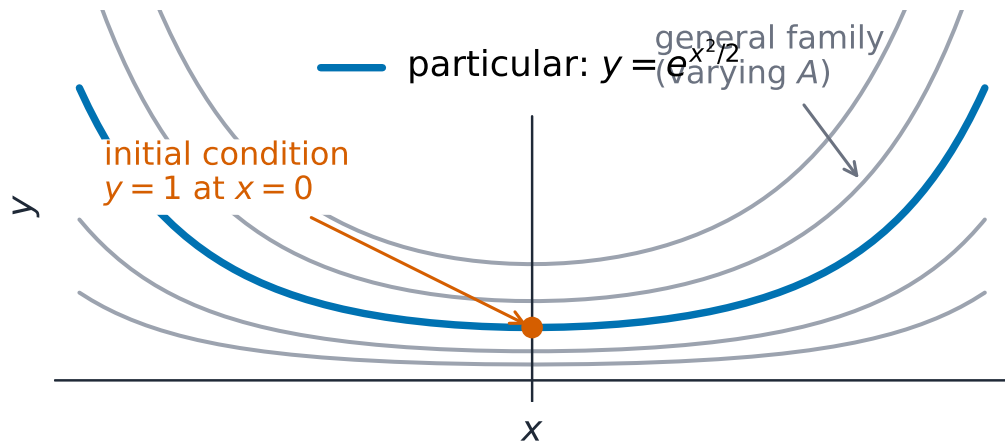
$$\frac{Ax + B}{x^2 + c} - \text{a linear numerator over a quadratic}$$

why bother

each piece integrates to a logarithm or a power, which the original fraction does not

Skipping the degree check is the commonest error, and it produces an answer that cannot be right because the shapes do not match.

Differential equations, in detail



general: family with A · IC → particular solution

$\frac{dy}{dx} = xy$ with $y = 1$ at $x = 0$ gives $\ln y = \frac{1}{2}x^2 + c$, so $y = e^{x^2/2}$. A condition fixes C (the **particular solution**).

Four more pieces of vocabulary

Integration by substitution

Integration by substitution
variable and the limits together.

reverses the chain rule – change the

Sign change and iteration

A **sign change**
an **iterative formula**
when $|g'| < 1$ nearby.

of a continuous f between a and b locates a root;
 $x_{n+1} = g(x_n)$ then **converges** to it

Cosecant and cotangent

$\operatorname{cosec} \theta = 1/\sin \theta$ and $\cot \theta = 1/\tan \theta$ – each undefined wherever its denominator vanishes.

Magnitude

The **magnitude**
the unit vector.

of a vector is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; dividing by it gives

State the interval, the two signs *and* the continuity. A sign change without continuity is an incomplete argument.

Exercises 2.1·2.2

2.1 Differentiate $y = \tan^{-1}(x^2)$. [2]

2.2 Differentiate $y = x \ln x$. [2]

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2.1

- $\frac{dy}{dx} = \frac{2x}{1+x^4}$

2.2 Differentiate $y = x \ln x$. [2]

2.2

- $\frac{dy}{dx} = \ln x + 1$

Exercise 3.1

3.1 $\frac{d}{dx} \tan^{-1}(3x)$ equals:

[1]

☐

A $\frac{3}{1 + 9x^2}$

B $\frac{1}{1 + 3x^2}$

C $\frac{3}{1 + 3x^2}$

D $\frac{1}{1 + 9x^2}$

Exercise 3.1

3.1 $\frac{d}{dx} \tan^{-1}(3x)$ equals:

[1]

- ☐ A $\frac{3}{1+9x^2}$
- ☐ B $\frac{1}{1+3x^2}$
- ☐ C $\frac{3}{1+3x^2}$
- ☐ D $\frac{1}{1+9x^2}$

3.1 A

$$\bullet \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$$

Exercise 3.2

3.2 A curve has parametric equations $x = 1 + 2 \cos t$, $y = 3 \sin t$ for $0 \leq t \leq \pi$. Find $\frac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$. [4]

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3.2

$$\bullet \quad \frac{dx}{dt} = -2 \sin t, \quad \frac{dy}{dt} = 3 \cos t; \quad \frac{dy}{dx} = \frac{3 \cos t}{-2 \sin t} = -\frac{3}{2} \cot t; \quad \text{at } t = \frac{\pi}{6}: \quad -\frac{3}{2} \cot 30^\circ = -\frac{3}{2} \sqrt{3} = -\frac{3\sqrt{3}}{2}$$

Exercise 3.3 (a)

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

[3]

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[3]

(a)

- $2xy + x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0; \frac{dy}{dx} = -\frac{2xy}{x^2 + 3y^2}$

Exercise 3.3 (b)

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(b) Find the gradient at the point $(1, 2)$.

[2]

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[2]

(b)

- at $(1, 2)$: $-\frac{2(1)(2)}{1 + 12} = -\frac{4}{13}$