

Exercise walk-through

2.4

Differentiation

A-Level Mathematics

What we will cover

- 1 Modulus & polynomials
- 2 Logarithms & exponentials
- 3 Trigonometry
- 4 Differentiation
- 5 Integration & numerical methods
- 6 Recap



Four techniques worth naming

Product rule

The **product rule** composition.

: $(uv)' = u'v + uv'$ – for a product, never for a

Integration by substitution

Integration by substitution reverses the chain rule: change the variable *and* the limits, or convert back at the end.

Natural logarithm

The **natural logarithm** c.

is the inverse of e^x , and $\int \frac{1}{x} dx = \ln |x| +$

Remainder theorem

The **remainder theorem** : the remainder when $f(x)$ is divided by $(x - a)$ is $f(a)$ – so a remainder of zero is the factor theorem.

Indices and **trigonometric identities** simplify first, integrate second. A polynomial's **degree**

are what make an integral doable: is its highest power.

Finding a root when the equation cannot be solved

a root

a solution of $f(x) = 0$ – and many equations have no exact form for one

the sign change rule

if $f(a)$ and $f(b)$ have opposite signs and f is continuous, a root lies between them

iteration

rearrange to $x = F(x)$, then use the **iterative formula**

$$x_{n+1} =$$

starting

begin from a value near the root, found by the sign change

converge

the sequence may converge to the root – or diverge, depending on the rearrangement chosen

the check

a sign change on a small interval either side confirms the root to the required accuracy

Different rearrangements of the *same* equation converge at different speeds, or not at all. The exam gives you the rearrangement to use.

Exercises 2.1·2.2

2.1 Differentiate $y = x^2 e^x$.

[2]

2.2 Differentiate $y = \frac{\sin x}{x}$.

[2]

Exercises 2.1·2.2

2.1 Differentiate $y = x^2 e^x$.

[2]

2.1

- $$\frac{dy}{dx} = 2xe^x + x^2 e^x = xe^x(2 + x)$$

2.2 Differentiate $y = \frac{\sin x}{x}$.

[2]

2.2

- $$\frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2}$$

Exercises 3.1·3.2

3.1 $\frac{d}{dx}(\ln(3x))$ equals:

[1]

☐

A $\frac{1}{3x}$

B $\frac{3}{x}$

C $\frac{1}{x}$

D $3\ln x$

3.2 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t , and the value of t at the stationary point(s).

[4]

Exercises 3.1·3.2

3.1 $\frac{d}{dx}(\ln(3x))$ equals:

[1]

☐

A $\frac{1}{3x}$

B $\frac{3}{x}$

C $\frac{1}{x}$

D $3\ln x$

3.1 C

- $\ln(3x) = \ln 3 + \ln x$, so the derivative is $\frac{1}{x}$

3.2 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t , and the value of t at the stationary point(s). [4]

3.2

- $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2 - 3$; $\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$; stationary where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$

Exercise 3.3 (a)

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

[3]

Exercise 3.3 (a)

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

[3]

(a)

- differentiate: $2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$; $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$

Exercise 3.3 (b)

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(b) Find the gradient of the curve at the point $(1, 2)$.

[2]

Exercise 3.3 (b)

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(b) Find the gradient of the curve at the point $(1, 2)$.

[2]

(b)

- at $(1, 2)$: $\frac{dy}{dx} = -\frac{2 + 2}{1 + 4} = -\frac{4}{5}$