

Exercise walk-through

2.3

Trigonometry

A-Level Mathematics

More trigonometry

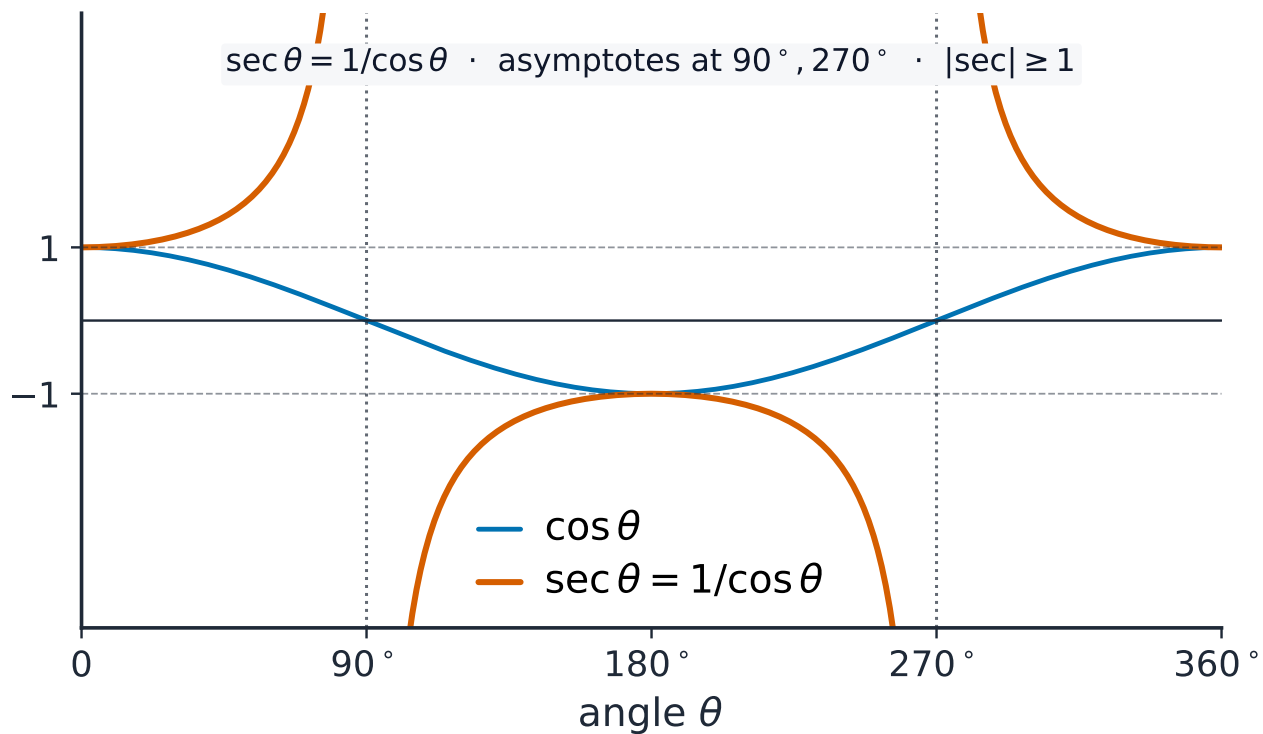
Reciprocals: $\sec = \frac{1}{\cos}$, $\csc = \frac{1}{\sin}$, $\cot = \frac{1}{\tan}$.

$$\sec^2 \theta \equiv 1 + \tan^2 \theta.$$

Compound & **double angle** ; **R-formula**

$$a \sin \theta + b \cos \theta = R \sin(\theta + \alpha).$$

More trigonometry, in detail



Reciprocals: $\sec = \frac{1}{\cos}$, $\csc = \frac{1}{\sin}$, $\cot = \frac{1}{\tan}$.

Worked example – reduce to one function first

Solve $2 \tan^2 \theta + 3 \sec \theta = 18$.

- replace $\tan^2 \theta$ with $\sec^2 \theta - 1$
- $2(\sec^2 \theta - 1) + 3 \sec \theta = 18$
- $2 \sec^2 \theta + 3 \sec \theta - 20 = 0$
- a quadratic in $\sec \theta$: factorise, then take $\cos \theta = 1/\sec \theta$

Always reduce to one trig function before solving.

The identity you need is the one that converts the squared term – here $\tan^2 \rightarrow \sec^2$.

The three reciprocal functions, and their identities

secant

$$\sec \theta = \frac{1}{\cos \theta}$$

cosecant

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

cotangent

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

from $\sin^2 + \cos^2 = 1$ divide by $\cos^2 \theta$: $1 + \tan^2 \theta \equiv \sec^2 \theta$

and by $\sin^2 \theta$

$$1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$$

compound angle

$\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$ – and the double-angle forms that follow

“co” pairs with *sine*, not cosine: **cosecant** is 1/sin. Getting that backwards is the standard slip.

Exercises 2.1·3.1

2.1 Write $\sec^2 \theta$ in terms of $\tan \theta$.

[1]

2.2 Use a double-angle formula to write $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.

[1]

3.1 Expressed as $R \sin(\theta + \alpha)$, the value of R for $3 \sin \theta + 4 \cos \theta$ is:

[1]

☐

A 5

B 7

C $\sqrt{7}$

D 12

Exercises 2.1·3.1

2.1 Write $\sec^2 \theta$ in terms of $\tan \theta$.

[1]

2.1

- $\sec^2 \theta = 1 + \tan^2 \theta$
-

2.2 Use a double-angle formula to write $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$.

[1]

2.2

- $\sin 2\theta = 2 \sin \theta \cos \theta$
-

3.1 Expressed as $R \sin(\theta + \alpha)$, the value of R for $3 \sin \theta + 4 \cos \theta$ is:

[1]

☐

A 5

B 7

C $\sqrt{7}$

D 12

3.1 A

- $R = \sqrt{3^2 + 4^2} = 5$

Exercise 3.2

3.2 Express $5 \sin \theta - 12 \cos \theta$ in the form $R \sin(\theta - \alpha)$ with $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

Exercise 3.2

3.2 Express $5 \sin \theta - 12 \cos \theta$ in the form $R \sin(\theta - \alpha)$ with $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

3.2

- $R = \sqrt{5^2 + 12^2} = 13$; $\tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.4^\circ$; $13 \sin(\theta - 67.4^\circ)$

Exercise 3.3

3.3 (a) Show that $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \equiv 2 \sec^2 \theta$. [3]

(b) Hence solve $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 8$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

Exercise 3.3

3.3 (a) Show that $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \equiv 2 \sec^2 \theta$. [3]

(b) Hence solve $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 8$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

3.3

(a)

- common denominator: $\frac{(1 + \sin \theta) + (1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \frac{2}{1 - \sin^2 \theta} = \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta$

(b)

- $2 \sec^2 \theta = 8 \Rightarrow \sec^2 \theta = 4 \Rightarrow \cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \pm \frac{1}{2}; \theta = 60^\circ \text{ or } 120^\circ$