

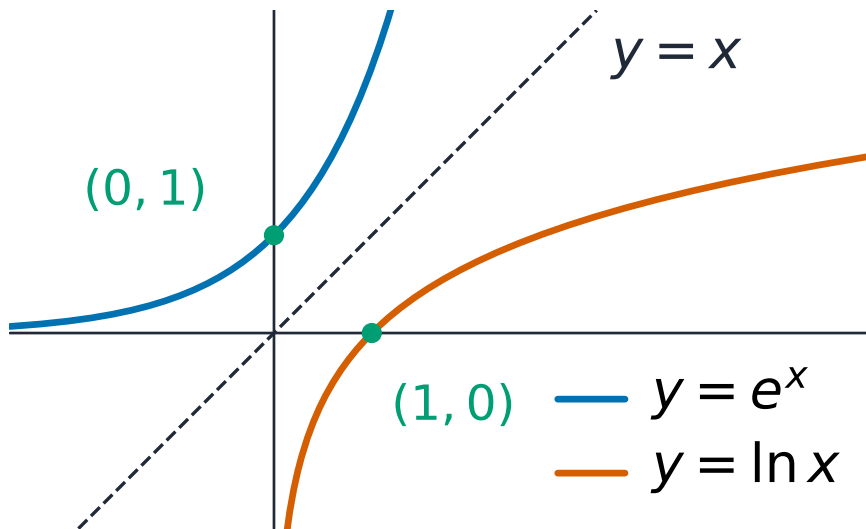
Exercise walk-through

2.2

Logarithmic and exponential functions

A-Level Mathematics

Logarithms and exponentials



Laws of logs :

$$\log mn = \log m + \log n,$$

$$\log m^k = k \log m.$$

e^x and $\ln x$ are inverses.

Unknown in the power $\Rightarrow$
take logs.

Linear form :

$$y = Ax^n \Rightarrow \ln y = \ln A + n \ln x.$$

ln and exp inverse · reflect in $y = x$

Exercises 2.1·2.2

2.1 Write $\log 12$ in terms of $\log 2$ and $\log 3$.

[2]

2.2 Solve $e^{2x} = 7$, giving x to 3 s.f.

[2]

Exercises 2.1·2.2

2.1 Write $\log 12$ in terms of $\log 2$ and $\log 3$.

[2]

2.1

- $12 = 2^2 \times 3$, so $\log 12 = 2\log 2 + \log 3$
-

2.2 Solve $e^{2x} = 7$, giving x to 3 s.f.

[2]

2.2

- $2x = \ln 7 \Rightarrow x = \frac{1}{2} \ln 7 = 0.973$

Exercises 3.1·3.2

3.1 The solution of $3^x = 20$ is $x =$

[1]

- ☐ **A** $\frac{\ln 20}{\ln 3}$
- B** $\frac{\ln 3}{\ln 20}$
- C** $\ln \frac{20}{3}$
- D** $20 - 3$

3.2 Solve the equation $\ln(2x + 1) + \ln 3 = \ln(4x + 9)$.

[3]

Exercises 3.1·3.2

3.1 The solution of $3^x = 20$ is $x =$

[1]

- ☐ **A** $\frac{\ln 20}{\ln 3}$
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3.1 A

- $x \ln 3 = \ln 20 \Rightarrow x = \frac{\ln 20}{\ln 3}$

3.2 Solve the equation $\ln(2x + 1) + \ln 3 = \ln(4x + 9)$.

[3]

3.2

- $\ln(3(2x + 1)) = \ln(4x + 9); 6x + 3 = 4x + 9 \Rightarrow 2x = 6; x = 3$

Exercise 3.3 (a)

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

(a) Find the values of n and A .

[3]

Exercise 3.3 (a)

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

(a) Find the values of n and A .

[3]

(a)

- gradient $= n = 1.5$; intercept $\ln A = 2 \Rightarrow A = e^2 = 7.39$

Exercise 3.3 (b)

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

(b) Find y when $x = 4$.

[2]

Exercise 3.3 (b)

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

(b) Find y when $x = 4$.

[2]

(b)

- $y = 7.39 \times 4^{1.5} = 7.39 \times 8 = 59.1$