

Exercise walk-through

# 1.8

## Integration

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A-Level Mathematics

# What we will cover

- 1 Quadratics & functions
- 2 Coordinate geometry
- 3 Circular measure & trigonometry
- 4 Series
- 5 Differentiation & integration
- 6 Recap



# What the derivative tells you about the shape

**Gradient of a curve**  $f'(x)$  is the **gradient of a curve** at a point – the gradient of the tangent there, as a **chord** shortens to nothing.

**Maximum and minimum** At a **maximum point**  $f' = 0$  and  $f'' < 0$ ; at a **minimum point**  $f' = 0$  and  $f'' > 0$ .

**Point of inflexion** A **point of inflexion** is where the curvature changes sign. A stationary one has  $f' = 0$  and  $f'' = 0$  – but  $f'' = 0$  alone is not enough.

**Integration** Reversing the process needs the **constant of integration** : one condition fixes it, and without it the answer is a family of curves.

Test a stationary point with the second derivative *or* with the sign of  $f'$  either side. Both are accepted; neither may be skipped.

## Exam tips

- Show **every line of algebra** – method marks are lost by jumping steps; use the **discriminant**  $b^2 - 4ac$  to decide the number of real roots.
- Work in **radians** for circular measure (arc =  $r\theta$ , sector area =  $\frac{1}{2}r^2\theta$ ) and for calculus of trig functions.
- For **differentiation**, set  $\frac{dy}{dx} = 0$  for stationary points and use the second derivative to classify them.
- For **integration**, add  $+c$  to an indefinite integral and use limits for area; an area below the axis gives a negative integral.

## Exercises 2.1·2.2

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**2.1** Find  $\int (6x^2 - 4x + 1) dx$ . [2]

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**2.2** Evaluate  $\int_1^2 \frac{1}{x^2} dx$ . [2]

## Exercises 2.1·2.2

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**2.1** Find  $\int (6x^2 - 4x + 1) dx$ .

[2]

**2.1**

- $2x^3 - 2x^2 + x + C$

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**2.2** Evaluate  $\int_1^2 \frac{1}{x^2} dx$ .

[2]

**2.2**

- $\left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$

## Exercises 3.1·3.2

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**3.1**  $\int_0^{\phantom{0}} 2x \, dx$  equals:

[1]

☐**A** 6**B** 9**C** 12**D** 18

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**3.2** A curve has  $\frac{dy}{dx} = 3x^2 - 4x$  and passes through  $(2, 5)$ . Find the equation of the curve.

[4]

## Exercises 3.1·3.2

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3.1  $\int_0^{\phantom{0}} 2x \, dx$  equals:

[1]

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A 6

B 9

C 12

D 18

3.1 B

- $[x^2]_0^3 = 9$

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3.2 A curve has  $\frac{dy}{dx} = 3x^2 - 4x$  and passes through  $(2, 5)$ . Find the equation of the curve. [4]

3.2

- $y = \int (3x^2 - 4x) \, dx = x^3 - 2x^2 + C$ ; use  $(2, 5)$ :  $5 = 8 - 8 + C \Rightarrow C = 5$ ;  $y = x^3 - 2x^2 + 5$

## Exercise 3.3 (a)

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**3.3** The region  $R$  is bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis and the line  $x = 4$ .

(a) Find the area of  $R$ .

[3]

## Exercise 3.3 (a)

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**3.3** The region  $R$  is bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis and the line  $x = 4$ .

(a) Find the area of  $R$ .

[3]

(a)

- $$\int_0^4 x^{1/2} dx = \left[ \frac{2}{3} x^{3/2} \right]_0^4 = \frac{2}{3}(8) = \frac{16}{3}$$

## Exercise 3.3 (b)

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**3.3** The region  $R$  is bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis and the line  $x = 4$ .

(b) Find the volume when  $R$  is rotated through  $360^\circ$  about the  $x$ -axis. [3]

## Exercise 3.3 (b)

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**3.3** The region  $R$  is bounded by the curve  $y = \sqrt{x}$ , the  $x$ -axis and the line  $x = 4$ .

(b) Find the volume when  $R$  is rotated through  $360^\circ$  about the  $x$ -axis. [3]

(b)

- $$V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x dx = \pi \left[ \frac{x^2}{2} \right]_0^4 = 8\pi$$