

Exercise walk-through

1.7

Differentiation

A-Level Mathematics

What we will cover

- 1 Quadratics & functions
- 2 Coordinate geometry
- 3 Circular measure & trigonometry
- 4 Series
- 5 Differentiation & integration
- 6 Recap



Integration

Reverse of **differentiation** $(+C)$:

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C.$$

$$\text{Area} = \int_p^q y dx.$$

Exam tips

- Show **every line of algebra** – method marks are lost by jumping steps; use the **discriminant** $b^2 - 4ac$ to decide the number of real roots.
- Work in **radians** for circular measure (arc = $r\theta$, sector area = $\frac{1}{2}r^2\theta$) and for calculus of trig functions.
- For **differentiation**, set $\frac{dy}{dx} = 0$ for stationary points and use the second derivative to classify them.
- For **integration**, add $+c$ to an indefinite integral and use limits for area; an area below the axis gives a negative integral.

Exercises 2.1·2.2

2.1 Differentiate $y = 3x^4 - \frac{2}{x}$. [2]

2.2 Find $\frac{dy}{dx}$ when $y = (3x + 1)^5$. [2]

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- $\frac{dy}{dx} = 12x^3 + \frac{2}{x^2}$

2.2 Find $\frac{dy}{dx}$ when $y = (3x + 1)^5$. [2]

2.2

- $\frac{dy}{dx} = 5(3x + 1)^4 \cdot 3 = 15(3x + 1)^4$

Exercise 3.1

3.1 The gradient of $y = x^3 - 2x$ at $x = 2$ is:

[1]

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A 4

B 6

C 10

D 12

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3.1 C

- $\frac{dy}{dx} = 3x^2 - 2$; at $x = 2$, $= 12 - 2 = 10$

Exercise 3.2 (a)

3.2 A curve has equation $y = x^3 - 3x^2 - 9x + 5$.

(a) Find the coordinates of the two stationary points.

[4]

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[4]

(a)

- $\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x-3)(x+1) = 0$; $x = 3$ or $x = -1$; $y(3) = 27 - 27 - 27 + 5 = -22$
and $y(-1) = -1 - 3 + 9 + 5 = 10$, so $(3, -22)$ and $(-1, 10)$

Exercise 3.2 (b)

3.2 A curve has equation $y = x^3 - 3x^2 - 9x + 5$.

(b) Use the second derivative to determine the nature of each, and hence write down the set of values of x for which the curve is decreasing. [3]

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(b)

- $\frac{d^2y}{dx^2} = 6x - 6$; at $x = 3$ it is $12 > 0$
- minimum, at $x = -1$ it is $-12 < 0$
- maximum; the curve falls between the maximum and the minimum, so it is decreasing for $-1 < x < 3$

Exercise 3.3

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3.3

- $\frac{dy}{dx} = -\frac{8}{x^2}$; at $x = 2$, gradient $= -2$; normal gradient $= \frac{1}{2}$; $y - 4 = \frac{1}{2}(x - 2) \Rightarrow y = \frac{1}{2}x + 3$