

Exercise walk-through

1.6

Series

A-Level Mathematics

Series

Binomial :

$$(a + b)^n = \sum \binom{n}{r} a^{n-r} b^r.$$

AP : $u_n = a + (n - 1)d.$

GP : $u_n = ar^{n-1}.$

Sum to infinity ($|r| < 1$)

$$S_{\infty} = \frac{a}{1 - r}.$$

e.g. $a = 32, r = -\frac{3}{4}$: $S_{\infty} = \frac{128}{7}.$

The binomial expansion

For a positive integer n ,

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + b^n,$$

$$\text{with } \binom{n}{r} = \frac{n!}{r! (n-r)!}.$$

First three terms of $(2 - px)^5$

$$32 - 80px + 80p^2x^2 + \cdots$$

Keep the sign with the term: $(-px)^2$ is **positive**.

Arithmetic and geometric progressions

AP

– adds a fixed **common difference** d :

$$u_n = a + (n - 1)d, \quad S_n = \frac{n}{2}(2a + (n - 1)d).$$

GP

– multiplies by a fixed **common ratio** r :

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1 - r^n)}{1 - r}.$$

The two progressions, and the binomial

Arithmetic progression

An **arithmetic progression** adds a common difference: $u_n = a + (n - 1)d$, and $S_n = \frac{n}{2}(2a + (n - 1)d)$.

Geometric progression

A **geometric progression** multiplies by a common ratio: $u_n = ar^{n-1}$, and the sum to infinity exists only when $|r| < 1$.

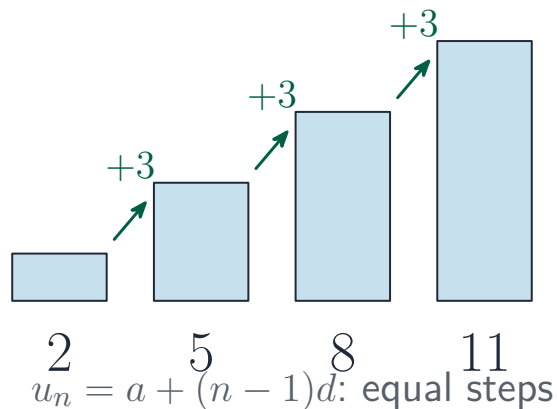
Binomial coefficient

The **binomial coefficient** $\binom{n}{r}$ counts the ways of choosing r from n – and it is the coefficient in the expansion of $(a + b)^n$.

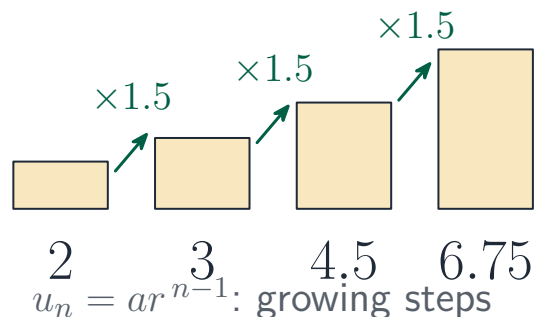
Check which progression before writing anything: a common *difference* or a common *ratio*, tested on the first three terms.

Arithmetic and geometric progressions, in detail

arithmetic: add d each step



geometric: multiply by r each step



An AP climbs in equal steps; a GP's steps grow by the same ratio

Exercises 2.1·2.2

2.1 Find the 10th term of the AP $3, 7, 11, \dots$ [2]

2.2 State the condition on r for a GP to have a sum to infinity. [1]

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2.1 Find the 10th term of the AP $3, 7, 11, \dots$ [2]

2.1

- $u_{10} = a + 9d = 3 + 9(4) = 39$

2.2 State the condition on r for a GP to have a sum to infinity. [1]

2.2

- $|r| < 1$

Exercise 3.1

3.1 The coefficient of x^2 in $(1 + 2x)^6$ is:

[1]

☐

A 12

B 30

C 60

D 15

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3.1 C

- $\binom{6}{2}(2)^2 = 15 \times 4 = 60$

Exercise 3.2 (a)

3.2 The first term of a GP is 24 and the sum to infinity is 40.

(a) Find the common ratio.

[3]

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(a) Find the common ratio.

[3]

(a)

- $$S_{\infty} = \frac{a}{1-r} = 40 \Rightarrow \frac{24}{1-r} = 40 \Rightarrow 1-r = 0.6; r = 0.4$$

Exercise 3.2 (b)

3.2 The first term of a GP is 24 and the sum to infinity is 40.

(b) Find the sum of the first three terms.

[2]

Exercise 3.2 (b)

3.2 The first term of a GP is 24 and the sum to infinity is 40.

(b) Find the sum of the first three terms.

[2]

(b)

- $$S_3 = \frac{24(1 - 0.4^3)}{1 - 0.4} = \frac{24(0.936)}{0.6} = 37.44$$

Exercise 3.3

3.3 An AP has first term 5 and common difference 3. The sum of the first n terms is 440. Find n . [4]

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3.3

- $S_n = \frac{n}{2}(2a + (n-1)d) = 440 \Rightarrow \frac{n}{2}(10 + 3(n-1)) = 440; 3n^2 + 7n - 880 = 0;$
 $(3n + 55)(n - 16) = 0; n = 16$ (reject negative)