

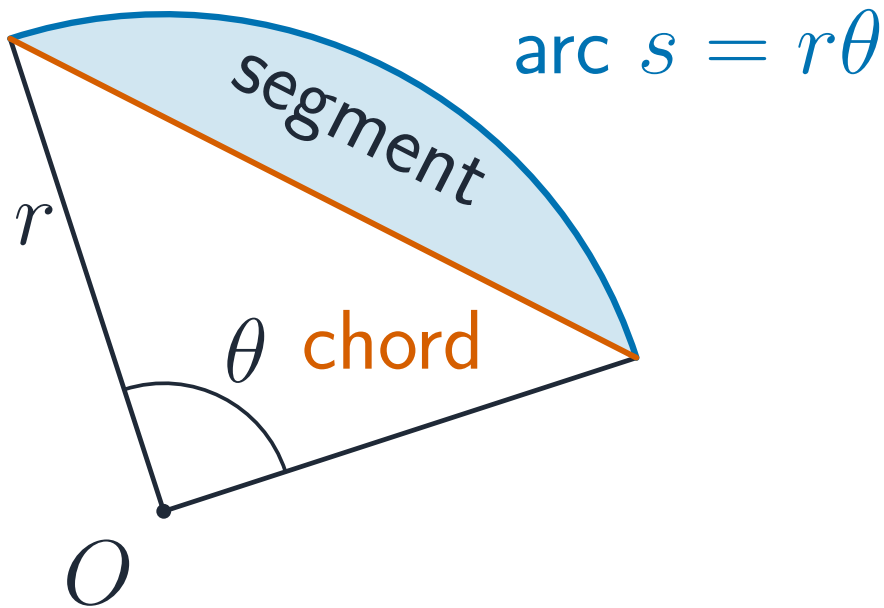
Exercise walk-through

1.5

Trigonometry

A-Level Mathematics

Circular measure



Radians : $\pi = 180^\circ$. For a
sector (angle in radians):

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta.$$

Segment = $\frac{1}{2}r^2(\theta - \sin \theta)$.

Radians, circles, and the trigonometric words

Degrees and radians

$180^\circ = \pi$ radians. Every calculus formula for a trigonometric function assumes radians, never **degrees**.

Circle parts

The **diameter** is twice the radius; a **chord** joins two points on the circumference; arc length is $r\theta$ and sector area $\frac{1}{2}r^2\theta$, with θ in radians.

The functions

Cosine and the **tangent function** join sine as the three ratios – and the tangent is undefined where the cosine is zero.

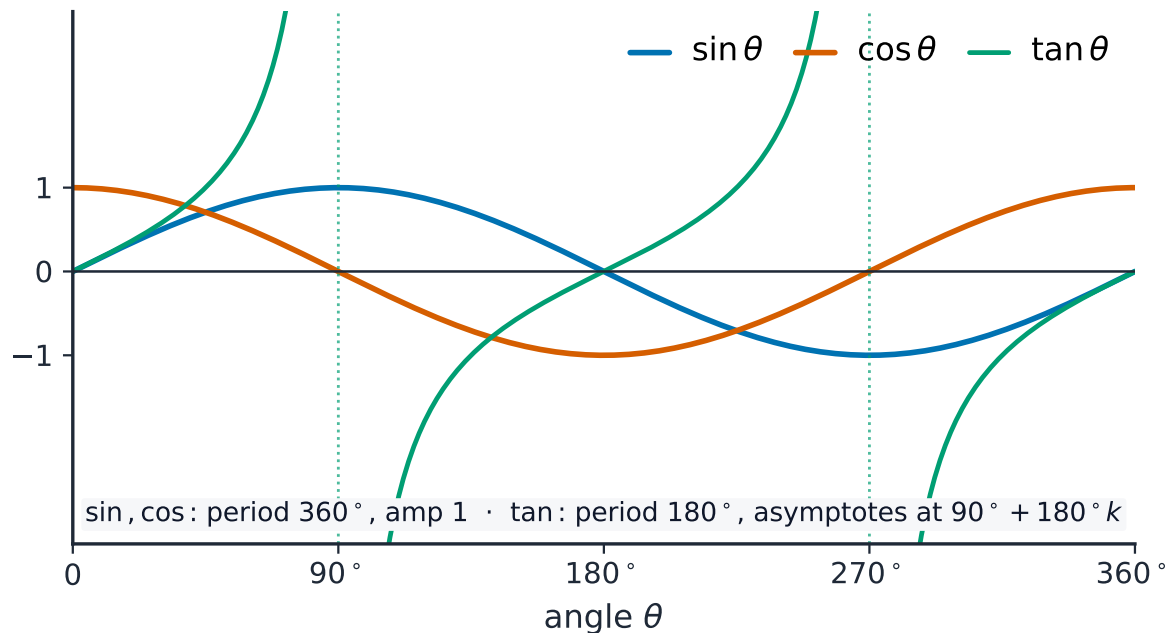
Principal value

An inverse trigonometric function returns only the **principal value**; the other solutions come from the graph or the CAST diagram.

The **equation of a straight line** topic to coordinate geometry.

$y - y_1 = m(x - x_1)$ and $\tan \theta = m$ connect this

Trigonometry



$\sin, \cos$ repeat every 360° ; learn exact values.

Identities :

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta},$$

$$\sin^2 \theta + \cos^2 \theta \equiv 1.$$

Reduce to one function, then solve in the interval.

Solving trigonometric equations

Solve $6 \sin \theta = 1 + \frac{2}{\sin \theta}$ **for** $-180^\circ < \theta < 180^\circ$.

Multiply by $\sin \theta$ – it is a **quadratic in** $\sin \theta$:

$$6 \sin^2 \theta - \sin \theta - 2 = 0 \Rightarrow (3 \sin \theta - 2)(2 \sin \theta + 1) = 0.$$

$$\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ; \quad \sin \theta = -\frac{1}{2} \Rightarrow \theta = -30^\circ, -150^\circ.$$

Reduce to **one** function first, then find **every** solution in the interval – the marks are lost by stopping at the first one.

Exercises 2.1·2.2

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Use an identity to write $\frac{\sin \theta}{\cos \theta}$ as a single function. [1]

Exercises 2.1·2.2

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.1

- $\frac{\sqrt{3}}{2}$

2.2 Use an identity to write $\frac{\sin \theta}{\cos \theta}$ as a single function. [1]

2.2

- $\tan \theta$

Exercises 3.1·3.2

3.1 The number of solutions of $\sin \theta = 0.4$ for $0^\circ \leq \theta \leq 360^\circ$ is:

[1]

☐

A 0

B 1

C 2

D 4

3.2 Solve $3 \cos^2 \theta + 2 \sin \theta - 3 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

[5]

Exercises 3.1·3.2

3.1 The number of solutions of $\sin \theta = 0.4$ for $0^\circ \leq \theta \leq 360^\circ$ is:

[1]

☐

A 0

B 1

C 2

D 4

3.1 C

- The sine curve gives two solutions in one full turn for $0 < 0.4 < 1$

3.2 Solve $3 \cos^2 \theta + 2 \sin \theta - 3 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

[5]

3.2

- use $\cos^2 \theta = 1 - \sin^2 \theta$: $3(1 - \sin^2 \theta) + 2 \sin \theta - 3 = 0 \Rightarrow -3 \sin^2 \theta + 2 \sin \theta = 0$;
 $\sin \theta(3 \sin \theta - 2) \cdot (-1) = 0 \Rightarrow \sin \theta = 0$ or $\sin \theta = \frac{2}{3}$; $\sin \theta = 0 \Rightarrow \theta = 0^\circ, 180^\circ, 360^\circ$;
 $\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ$

Exercise 3.3

3.3 Solve $2 \sin 2x = 1$ for $0^\circ \leq x \leq 180^\circ$.

[4]

Exercise 3.3

3.3 Solve $2 \sin 2x = 1$ for $0^\circ \leq x \leq 180^\circ$.

[4]

3.3

- $\sin 2x = \frac{1}{2}$; for $0^\circ \leq 2x \leq 360^\circ$, $2x = 30^\circ, 150^\circ$; $x = 15^\circ, 75^\circ$