

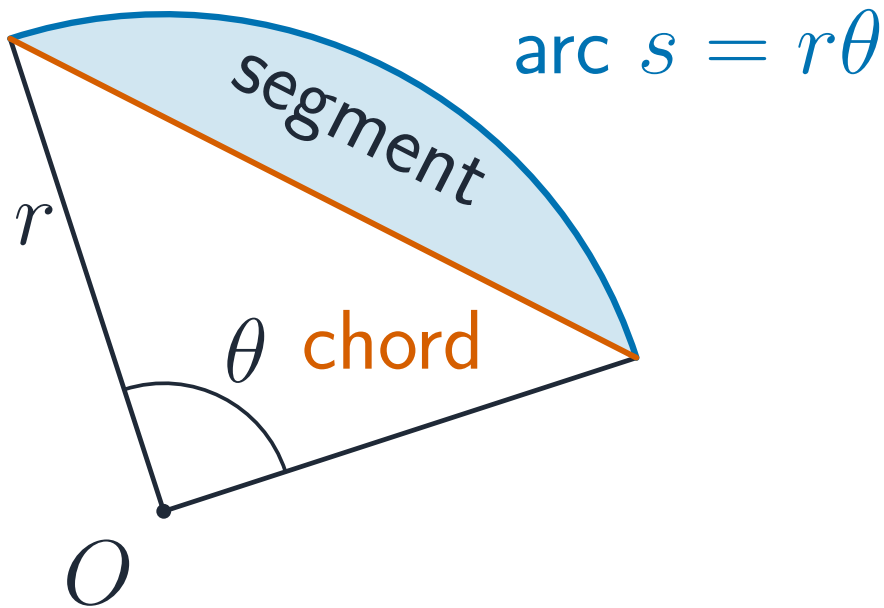
Exercise walk-through

1.4

Circular measure

A-Level Mathematics

Circular measure



Radians : $\pi = 180^\circ$. For a
sector (angle in radians):

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta.$$

Segment = $\frac{1}{2}r^2(\theta - \sin \theta)$.

Radians, circles, and the trigonometric words

Degrees and radians

$180^\circ = \pi$ radians. Every calculus formula for a trigonometric function assumes radians, never **degrees**.

Circle parts

The **diameter** is twice the radius; a **chord** joins two points on the circumference; arc length is $r\theta$ and sector area $\frac{1}{2}r^2\theta$, with θ in radians.

The functions

Cosine and the **tangent function** join sine as the three ratios – and the tangent is undefined where the cosine is zero.

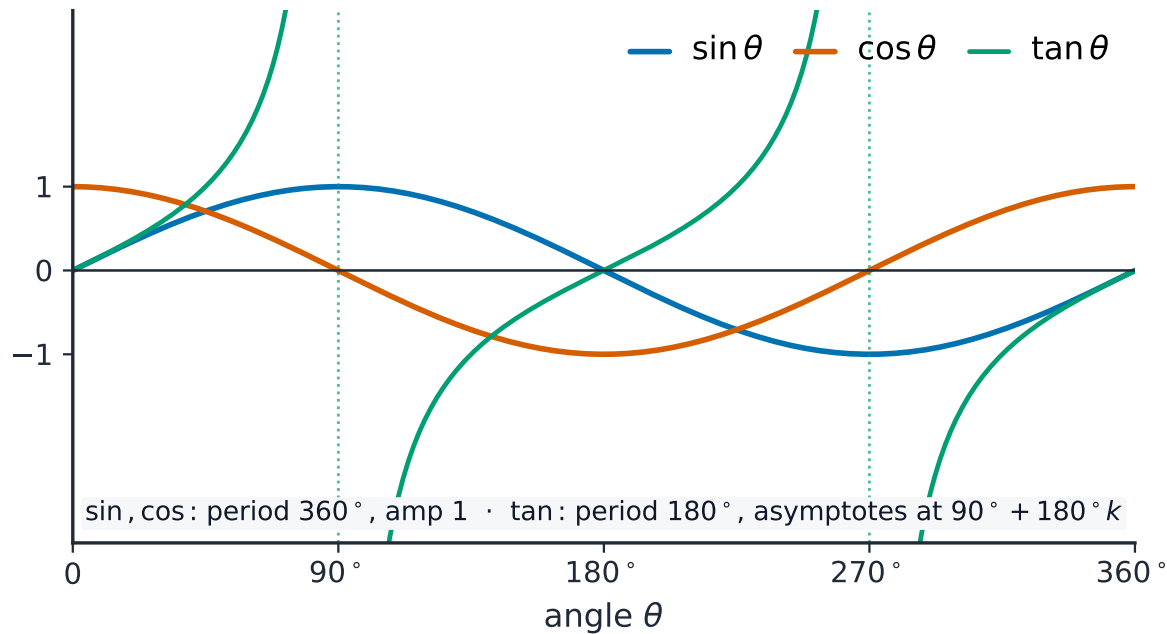
Principal value

An inverse trigonometric function returns only the **principal value**; the other solutions come from the graph or the CAST diagram.

The **equation of a straight line** topic to coordinate geometry.

$y - y_1 = m(x - x_1)$ and $\tan \theta = m$ connect this

Trigonometry



$\sin, \cos$ repeat every 360° ; learn exact values.

Identities :

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta},$$

$$\sin^2 \theta + \cos^2 \theta \equiv 1.$$

Reduce to one function, then solve in the interval.

Solving trigonometric equations

Solve $6 \sin \theta = 1 + \frac{2}{\sin \theta}$ **for** $-180^\circ < \theta < 180^\circ$.

Multiply by $\sin \theta$ – it is a **quadratic in** $\sin \theta$:

$$6 \sin^2 \theta - \sin \theta - 2 = 0 \Rightarrow (3 \sin \theta - 2)(2 \sin \theta + 1) = 0.$$

$$\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ; \quad \sin \theta = -\frac{1}{2} \Rightarrow \theta = -30^\circ, -150^\circ.$$

Reduce to **one** function first, then find **every** solution in the interval – the marks are lost by stopping at the first one.

Exercises 2.1·2.2

2.1 Convert 120° to radians (exact).

[1]

2.2 A sector has $r = 5$ cm and $\theta = 0.8$ rad. Find its arc length.

[1]

Exercises 2.1·2.2

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[1]

2.1

- $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

2.2 A sector has $r = 5$ cm and $\theta = 0.8$ rad. Find its arc length.

[1]

2.2

- $s = r\theta = 5 \times 0.8 = 4$ cm

Exercise 3.1

3.1 The area of a sector with $r = 4$ and $\theta = \frac{\pi}{6}$ is:

[1]

☐

A $\frac{2\pi}{3}$

B $\frac{4\pi}{3}$

C $\frac{\pi}{3}$

D 2π

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C $\frac{\pi}{3}$

D 2π

3.1 B

- $A = \frac{1}{2}(16)\frac{\pi}{6} = \frac{4\pi}{3}$

Exercise 3.2 (a)

3.2 A sector OAB has centre O , radius 10 cm and angle $AOB = 1.2$ radians.

(a) Find the perimeter of the sector.

[3]

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(a)

- arc = $10 \times 1.2 = 12$; perimeter = $12 + 10 + 10 = 32$ cm

Exercise 3.2 (b)

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(b) Find the area of the **segment** between the chord AB and the arc. [3]

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(b)

- $\text{segment} = \frac{1}{2}(10^2)(1.2 - \sin 1.2) = 50(1.2 - 0.9320) = 50(0.2680) = 13.4 \text{ cm}^2$

Exercise 3.3

3.3 A sector has perimeter 20 cm and area 16 cm^2 . Show that the radius r satisfies $r^2 - 10r + 16 = 0$, and hence find the two possible radii. [4]

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3.3

- perimeter: $2r + r\theta = 20$; area: $\frac{1}{2}r^2\theta = 16 \Rightarrow r\theta = \frac{32}{r}$. Substitute: $2r + \frac{32}{r} = 20 \Rightarrow 2r^2 - 20r + 32 = 0 \Rightarrow r^2 - 10r + 16 = 0$. Factorise $(r - 2)(r - 8) = 0 \Rightarrow r = 2$ or $r = 8$