

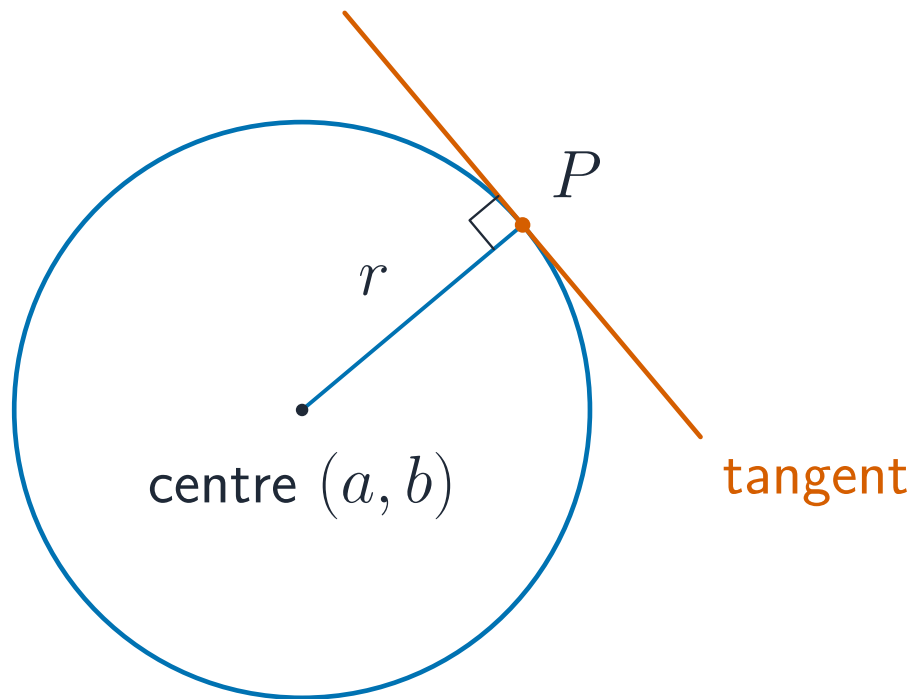
Exercise walk-through

1.3

Coordinate geometry

A-Level Mathematics

Coordinate geometry



Line gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$;
perpendicular $\Rightarrow m_1 m_2 = -1$.

$$(x - a)^2 + (y - b)^2 = r^2.$$

A **tangent** meets the **radius** at a right angle.

Straight lines

The **gradient** through (x_1, y_1) and (x_2, y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Three forms of the same line:

$$y = mx + c, \quad y - y_1 = m(x - x_1), \quad ax + by + c = 0.$$

Parallel : equal gradients.

Perpendicular : the gradients multiply to -1 .

Circles

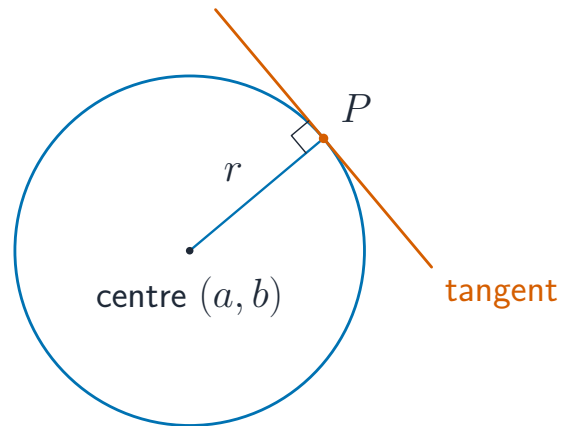
Centre (a, b) , radius r :

$$(x - a)^2 + (y - b)^2 = r^2.$$

An expanded form such as

$x^2 + y^2 - 6x + 10y - 27 = 0$ is the **same circle** – complete the square in x and in y to read off the centre and radius.

A **tangent** meets the radius at a **right angle**. That one fact solves most circle problems.



Exercises 2.1·2.2

2.1 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.2 Write the centre and radius of $(x - 3)^2 + (y + 2)^2 = 25$. [2]

Exercises 2.1·2.2

2.1 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.1

- $$m = \frac{7 - (-1)}{6 - 2} = \frac{8}{4} = 2$$

2.2 Write the centre and radius of $(x - 3)^2 + (y + 2)^2 = 25$. [2]

2.2

- centre $(3, -2)$; radius 5

Exercise 3.1

3.1 The line through $(0, 4)$ perpendicular to $y = \frac{1}{3}x + 1$ has gradient:

[1]

☐

A $\frac{1}{3}$

B 3

C -3

D $-\frac{1}{3}$

Exercise 3.1

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3.1 C

- Perpendicular gradient $= -1 \div \frac{1}{3} = -3$

Exercise 3.2 (a)

3.2 A circle has equation $x^2 + y^2 + 6x - 10y + 9 = 0$.

(a) Find its centre and radius.

[3]

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[3]

(a)

- complete the square: $(x + 3)^2 - 9 + (y - 5)^2 - 25 + 9 = 0 \Rightarrow (x + 3)^2 + (y - 5)^2 = 25$;
centre $(-3, 5)$, radius 5

Exercise 3.2 (b)

3.2 A circle has equation $x^2 + y^2 + 6x - 10y + 9 = 0$.

(b) The line $y = 5$ meets the circle at two points. Find their coordinates, and state what this chord is called. [3]

Exercise 3.2 (b)

3.2 A circle has equation $x^2 + y^2 + 6x - 10y + 9 = 0$.

(b) The line $y = 5$ meets the circle at two points. Find their coordinates, and state what this chord is called. [3]

(b)

- put $y = 5$: $(x + 3)^2 + 0 = 25$, so $x + 3 = \pm 5$; $x = 2$ or $x = -8$, giving $(2, 5)$ and $(-8, 5)$; $y = 5$ passes through the centre, so the chord is a **diameter** – and its length, 10, is indeed $2r$.

Exercise 3.3 (a)

3.3 The points $A(1, 2)$ and $B(5, 10)$ are given.

(a) Find the equation of the perpendicular bisector of AB .

[4]

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[4]

(a)

- midpoint $(3, 6)$; gradient $AB = \frac{10 - 2}{5 - 1} = 2$, so bisector gradient $= -\frac{1}{2}$; $y - 6 = -\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{15}{2}$

Exercise 3.3 (b)

3.3 The points $A(1, 2)$ and $B(5, 10)$ are given.

(b) The perpendicular bisector meets the x -axis at C . Find the coordinates of C . [2]

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(b) The perpendicular bisector meets the x -axis at C . Find the coordinates of C . [2]

(b)

- set $y = 0$: $0 = -\frac{1}{2}x + \frac{15}{2} \Rightarrow x = 15$, so $C(15, 0)$