

Exercise walk-through

1.2

Functions

A-Level Mathematics

Quadratics

$ax^2 + bx + c = 0$ has discriminant $\Delta = b^2 - 4ac$.

$$\Delta > 0$$

two distinct real roots

$$\Delta = 0$$

one repeated real root

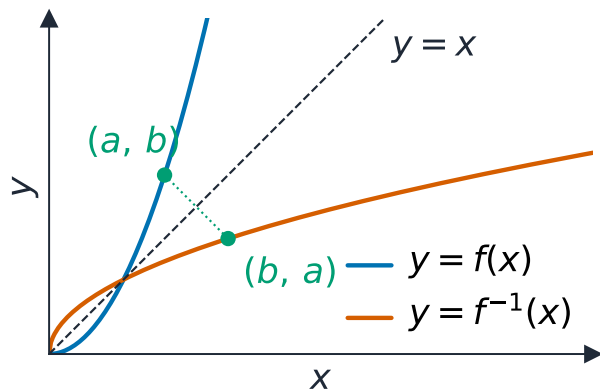
$$\Delta < 0$$

no real roots

Functions

- **domain** (inputs), **range** (outputs)
- **one-one** $\Rightarrow$ has an **inverse**
- **composition** $fg(x) = f(g(x))$
- **transformations** : $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$

Functions, in detail



inverse = reflection in $y = x$ · domain of f^{-1} = range of f

f^{-1} reflects f in $y = x$

Reflecting $y = f(x)$ in the line $y = x$ gives its inverse; a point (a, b) becomes (b, a) .

The vocabulary of a function question

Composition of functions

$fg(x)$ means do g first: the **composition of functions**, and the order matters.

Increasing and decreasing

An **increasing function** has $f'(x) > 0$ throughout an interval; a **decreasing function** has $f'(x) < 0$.

Identity

An **identity** is true for every value of x – unlike an equation, which is true for some.

Coefficients

The **coefficients inequality** are the numbers multiplying each power; a **quadratic** is solved from the sketch, not by dividing through.

An inverse exists only for a one-one function – which is why a domain is so often restricted before an inverse is asked for.

Finding an inverse

To find f^{-1} : write $y = f(x)$, make x the subject, then swap the letters.

The domain of f decides which square root you keep.

$f(x) = (x + 3)^2 - 12$ for $x \geq 0$. Find f^{-1} .

$(x + 3)^2 = y + 12 \Rightarrow x + 3 = \sqrt{y + 12}$,
taking the **positive** root because $x \geq 0$ means
 $x + 3 \geq 3 > 0$.

$$f^{-1}(x) = \sqrt{x + 12} - 3.$$

Exercises 2.1·2.2

2.1 $f(x) = 2x - 5$ and $g(x) = x^2$. Find $fg(3)$. [2]

2.2 Describe the single transformation mapping $y = f(x)$ to $y = f(x) - 4$. [1]

Exercises 2.1·2.2

2.1 $f(x) = 2x - 5$ and $g(x) = x^2$. Find $fg(3)$. [2]

2.1

- $g(3) = 9$; $f(9) = 2(9) - 5 = 13$

2.2 Describe the single transformation mapping $y = f(x)$ to $y = f(x) - 4$. [1]

2.2

- a translation **down** by 4

Exercise 3.1

3.1 $f(x) = 3 - 2x$. Then $f^{-1}(x)$ is:

[1]

☐

A $\frac{3-x}{2}$

B $\frac{x-3}{2}$

C $2x-3$

D $\frac{3+x}{2}$

Exercise 3.1

3.1 $f(x) = 3 - 2x$. Then $f^{-1}(x)$ is:

[1]

- ☐ **A** $\frac{3-x}{2}$
B $\frac{x-3}{2}$
C $2x-3$
D $\frac{3+x}{2}$

3.1 A

- $y = 3 - 2x \Rightarrow x = \frac{3-y}{2}$, so $f^{-1}(x) = \frac{3-x}{2}$

Exercise 3.2 (a)

3.2 The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(a) Express $f(x)$ in completed-square form and state the range of f .

[3]

Exercise 3.2 (a)

3.2 The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(a) Express $f(x)$ in completed-square form and state the range of f .

[3]

(a)

- $x^2 - 6x + 5 = (x - 3)^2 - 4$; for $x \geq 3$, range is $f(x) \geq -4$

Exercise 3.2 (b)

3.2 The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(b) Find $f^{-1}(x)$ and state its domain.

[3]

Exercise 3.2 (b)

3.2 The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

(b) Find $f^{-1}(x)$ and state its domain.

[3]

(b)

- $y = (x - 3)^2 - 4 \Rightarrow x = 3 + \sqrt{x + 4}$
- $f^{-1}(x) = 3 + \sqrt{x + 4}$; domain $x \geq -4$

Exercise 3.3

3.3 The graph of $y = f(x)$ is transformed to $y = 2f(x + 1)$. Describe the **two** transformations, in the correct order. [2]

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3.3

- translation left by 1 ($x \rightarrow x + 1$); then a stretch in the y -direction, scale factor 2