

Exercise walk-through

1.1

Quadratics

A-Level Mathematics

Quadratics

$ax^2 + bx + c = 0$ has discriminant $\Delta = b^2 - 4ac$.

$$\Delta > 0$$

two distinct real roots

$$\Delta = 0$$

one repeated real root

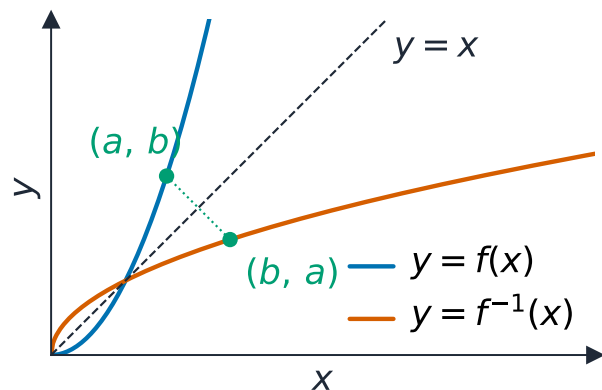
$$\Delta < 0$$

no real roots

Functions

- **domain** (inputs), **range** (outputs)
- **one-one** $\Rightarrow$ has an **inverse**
- **composition** $fg(x) = f(g(x))$
- **transformations** : $f(x) + a$, $f(x + a)$, $af(x)$, $f(ax)$

Functions, in detail



inverse = reflection in $y = x$ · domain of f^{-1} = range of f

f^{-1} reflects f in $y = x$

Reflecting $y = f(x)$ in the line $y = x$ gives its inverse; a point (a, b) becomes (b, a) .

The vocabulary of a function question

Composition of functions

$fg(x)$ means do g first: the **composition of functions**, and the order matters.

Increasing and decreasing

An **increasing function** has $f'(x) > 0$ throughout an interval; a **decreasing function** has $f'(x) < 0$.

Identity

An **identity** is true for every value of x – unlike an equation, which is true for some.

Coefficients

The **coefficients inequality** are the numbers multiplying each power; a **quadratic** is solved from the sketch, not by dividing through.

An inverse exists only for a one-one function – which is why a domain is so often restricted before an inverse is asked for.

Finding an inverse

To find f^{-1} : write $y = f(x)$, make x the subject, then swap the letters.

The domain of f decides which square root you keep.

$f(x) = (x + 3)^2 - 12$ for $x \geq 0$. Find f^{-1} .

$(x + 3)^2 = y + 12 \Rightarrow x + 3 = \sqrt{y + 12}$,
taking the **positive** root because $x \geq 0$ means
 $x + 3 \geq 3 > 0$.

$$f^{-1}(x) = \sqrt{x + 12} - 3.$$

Exercises 2.1·2.2

2.1 Express $x^2 + 8x + 3$ in the form $(x + p)^2 + q$. [2]

2.2 State the condition on the discriminant for a quadratic to have **no** real roots. [1]

Exercises 2.1·2.2

2.1 Express $x^2 + 8x + 3$ in the form $(x + p)^2 + q$. [2]

2.1

- $(x + 4)^2 - 13$

2.2 State the condition on the discriminant for a quadratic to have **no** real roots. [1]

2.2

- $b^2 - 4ac < 0$

Exercises 3.1·3.2

3.1 The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is: [1]

☐

A 0

B 1

C 2

D 4

3.2 Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$ and hence write down the least value of the expression. [4]

Exercises 3.1·3.2

3.1 The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is: [1]

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3.1 C

- Repeated root needs $b^2 - 4ac = 0$: $16 - 8c = 0 \Rightarrow c = 2$

3.2 Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$ and hence write down the least value of the expression. [4]

3.2

- $3(x^2 + 4x) + 7 = 3((x + 2)^2 - 4) + 7 = 3(x + 2)^2 - 5$; least value = -5 (when $x = -2$)

Exercises 3.3·3.4

3.3 Find the set of values of k for which the line $y = kx - 2$ meets the curve $y = x^2 - 3x + 2$ at two distinct points. [4]

3.4 Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$. [4]

Exercises 3.3·3.4

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3.3

- set equal: $x^2 - 3x + 2 = kx - 2 \Rightarrow x^2 - (3 + k)x + 4 = 0$; two points need $b^2 - 4ac > 0$:
 $(3 + k)^2 - 16 > 0$; $(k + 7)(k - 1) > 0$; $k < -7$ or $k > 1$
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3.4 Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$. [4]

3.4

- substitute: $x^2 + (2x - 1)^2 = 2 \Rightarrow 5x^2 - 4x - 1 = 0$; $(5x + 1)(x - 1) = 0 \Rightarrow x = -\frac{1}{5}$ or $x = 1$; $y = 2x - 1$ gives $(1, 1)$ and $(-\frac{1}{5}, -\frac{7}{5})$