HOLIDAY HOMEWORK 假期作业

AS-Level Mathematics

Exercise sheets and past-paper practice, topic by topic.

每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

Contents 目录

1 Pure Mathematics 1 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
factorising	因式分解			
parabola	抛物线			
quadratic	二次式			
coefficients	系数			
Completing the square	配方			
vertex	顶点			
discriminant	判别式			
real roots	实根			

Recall

At IGCSE you worked with quadratics. Bring this with you:

- You solved quadratic equations by **factorising** 因式分解.
- You drew the **parabola** 抛物线—the U-shaped curve of $y = ax^2 + bx + c$.

At AS Pure 1 we add a tidy form that reveals the curve's turning point, and a quick test for how many solutions an equation has.

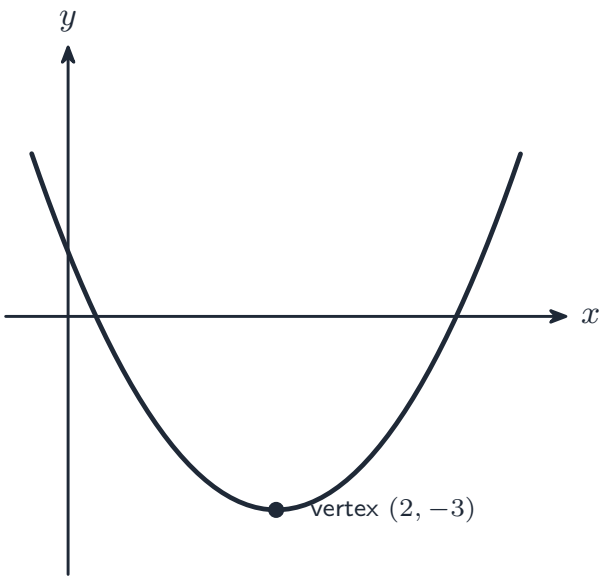
New ideas

■ 1 Completing the square

A **quadratic** 二次式 has the form ax^2+bx+c; the numbers a, b, c are its **coefficients** 系数.

Completing the square 配方 rewrites it as $a(x+p)^2+q$. This form shows the **vertex** 顶点 (turning point) of the curve at $(-p,\ q)$.

Worked example. $x^2-4x+1=(x-2)^2-4+1=(x-2)^2-3$. The vertex is at $(2,-3)$.

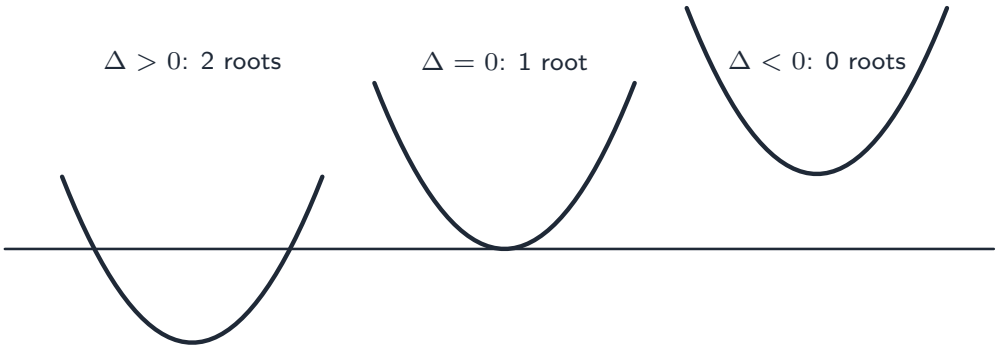


Micro-check. Write x^2-6x in the form $(x+p)^2+q$. [1]

■ 2 The discriminant

The **discriminant** 判别式 of ax^2+bx+c is $\Delta=b^2-4ac$. It tells you how many **real roots** 实根 (real solutions) the equation $ax^2+bx+c=0$ has:

- $b^2-4ac>0$ —two real roots;
- $b^2-4ac=0$ —one (repeated) real root;
- $b^2-4ac<0$ —no real roots.



Worked example. For $x^2 - 4x + 1$: $\Delta = (-4)^2 - 4(1)(1) = 16 - 4 = 12 > 0$, so there are **two** real roots.

Micro-check. Find the discriminant of $x^2 + 2x + 5$, and state the number of real roots. [2]

Watch out: to complete the square, halve b and square it. In $a(x+p)^2+q$ the vertex is at $(-p,\ q)$ —note the *sign flip* on p. For the discriminant: $b^2 - 4ac > 0 \to$ two roots, $= 0 \to$ one, $< 0 \to$ none.

Practice

1.1 Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$. [2]

1.2 State the vertex of the curve $y = (x - 3)^2 - 4$. [1]

1.3 Find the discriminant of $2x^2 + 3x + 1$, and state how many real roots the equation $2x^2 + 3x + 1 = 0$ has. [2]

1.4 State the number of real roots of $x^2 - 4x + 4 = 0$ (use the discriminant). [2]

1.5 Solve $x^2 - 5x + 6 = 0$ by factorising. [2]

1.6 Challenge. Find the values of k for which $x^2 + kx + 9 = 0$ has exactly one (repeated) real root. [3]

4 **(a)** Express $1-6x-x^2$ in the form $a-(x+b)^2$, where a and b are constants. [3]

(b) The graph of $y=x^2$ is transformed to the graph of $y=1-6x-x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n. [3]

6 Functions f and g are defined by

$$\mathrm{f}(x) = (x+3)^2 - 12 \quad \text{for } x \geqslant 0,$$

$$\mathrm{g}(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

(a) State the range of f. [1]

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(b) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

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(c) Solve the equation $\mathrm{gf}(x) = 69$. [4]

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10 A circle has equation $x^2+y^2+4y-21=0$ and a straight line has equation $2x+y-8=0$. The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection. [4]

(b) The circle has centre C and the two points of intersection are denoted by A and B.

Find the area of the triangle ABC. [3]

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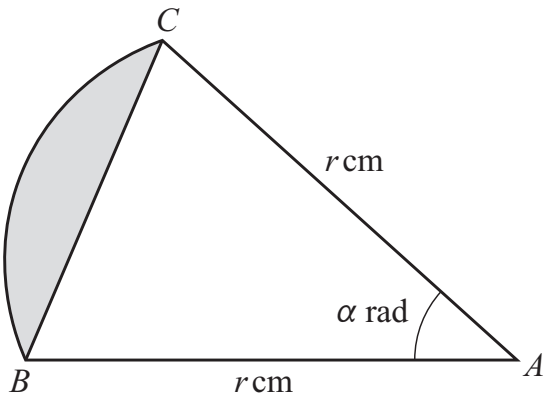

The diagram shows a sector ABC of a circle with centre A and radius r cm. The angle BAC is α radians, where $0 < \alpha < \frac{1}{2}\pi$.

(a) It is given that the area of the triangle ABC is $4\,\text{cm}^2$ and the area of the sector ABC is $8\alpha\,\text{cm}^2$.

Find the exact area of the shaded segment. [4]

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(b) It is given instead that the length of the chord BC is $\frac{1}{\sqrt{2}}r$ cm but the area of the triangle ABC is still $4\,\text{cm}^2$.

Find the area of the shaded segment. Give your answer correct to 3 significant figures. [4]

1 Pure Mathematics 1 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
gradient	斜率			
parallel	平行			
perpendicular	垂直			
circle	圆			
centre	圆心			
radius	半径			
tangent	切线			

Recall

In **Part 1** you completed the square and used the discriminant. Also bring your IGCSE work:

- you found the **gradient** of a line and used $y = mx + c$;
- you plotted points (x, y) on axes.

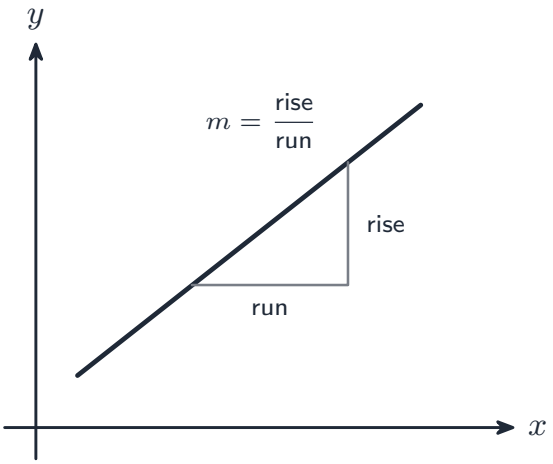
Part 2 sharpens straight lines (parallel and perpendicular) and adds the equation of a **circle**.

New ideas

■ 1 Straight lines

The **gradient** 斜率 (steepness) of the line joining (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$



Two lines are **parallel** 平行 when their gradients are **equal**, and **perpendicular** 垂直 (at right angles) when the product of their gradients is -1.

Worked example. A line has gradient 2. A line perpendicular to it has gradient $-\frac{1}{2}$ (because $2 \times -\frac{1}{2} = -1$).

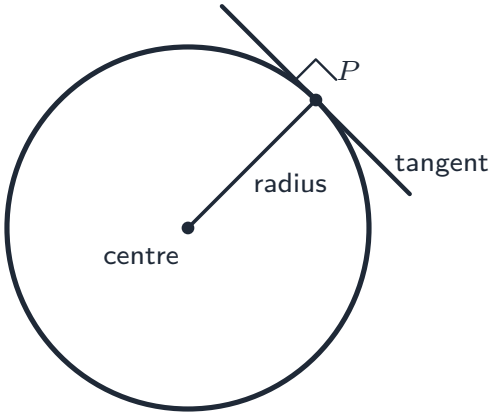
Micro-check. A line has gradient 3. State the gradient of a line **perpendicular** to it. [1]

■ 2 Circles

A **circle** 圆 with **centre** 圆心 (a, b) and **radius** 半径 r has the equation

$$(x - a)^2 + (y - b)^2 = r^2.$$

A **tangent** 切线 touches the circle at one point and is **perpendicular** to the radius drawn to that point.



Worked example. A circle with centre $(1, 2)$ and radius 3 has equation $(x-1)^2+(y-2)^2=9$.

Micro-check. Write the equation of the circle with centre $(0, 0)$ and radius 5. [1]

Watch out: parallel lines have *equal* gradients; perpendicular lines have gradients whose *product* is -1 (so m and $-1/m$). For $(x-a)^2+(y-b)^2=r^2$ the centre is (a, b) —note the *sign flips* —and the radius is r, not r^2.

Practice

1.1 Find the gradient of the line joining $(1, 2)$ and $(5, 10)$. [2]

1.2 A line has gradient $\frac{1}{2}$. State the gradient of a line (a) parallel to it, (b) perpendicular to it. [2]

1.3 Write the equation of the circle with centre $(3, -1)$ and radius 4. [2]

1.4 State the centre and radius of the circle $(x - 2)^2 + (y + 5)^2 = 49$. [2]

1.5 A circle has centre $(4, 6)$ and passes through the origin $(0, 0)$. Find its radius. [2]

1.6 Challenge. Find the equation of the line through $(2, 3)$ that is perpendicular to $y = 2x + 1$. [3]

5 **(a)** Show that $\tan^4\theta - 1 \equiv \dfrac{1 - 2\cos^2\theta}{\cos^4\theta}$. [3]

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(b) Hence solve the equation $\cos^2\theta(\tan^4\theta - 1) = 7$ for $0° < \theta < 180°$. [4]

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3 The coefficient of x^{7} in the expansion of $\left(px^{2}+\frac{1}{px}\right)^{8}$ is 1280.

Find the value of the constant p. [4]

4 A point P is moving along the curve with equation $y = ax^{\frac{5}{2}} - 12x$ in such a way that the x-coordinate of P is increasing at a constant rate of 5 units per second.

(a) Find the rate at which the y-coordinate of P is changing when $x = 9$. Give your answer in terms of the constant a. [3]

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(b) Given that the curve has a minimum point when $x = \frac{1}{4}$, find the value of a. [2]

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4 The equation of a curve is such that $\frac{dy}{dx} = kx^3 + \frac{2}{x^2}$, where k is a constant. The curve passes through the point S (2, 20) and the gradient of the curve at S is $\frac{65}{2}$.

(a) Find the value of k. [1]

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(b) The coordinates of a point T on the curve are $(1, t)$.

Find the value of t. [5]

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2 Pure Mathematics 2 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
modulus	绝对值			
polynomial	多项式			
degree	次数			
remainder theorem	余数定理			
remainder	余数			
factor theorem	因式定理			

Recall

At IGCSE you worked with algebra and the size of a number. Bring this with you:

- the size of a number ignoring its sign (e.g. the "distance from zero");
- factorising and dividing simple expressions.

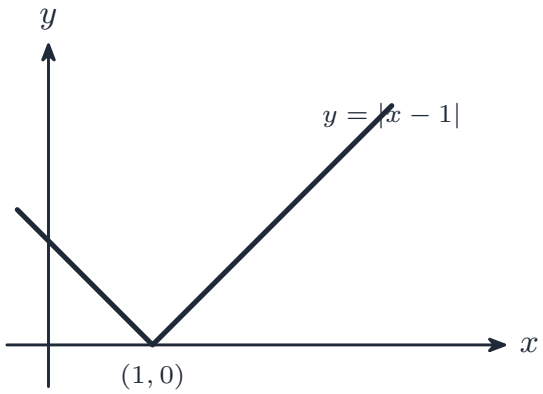
Pure 2 adds the **modulus** as a function, and a quick way to test factors of bigger polynomials.

New ideas

■ 1 The modulus

The **modulus** 绝对值 $|x|$ is the size of a number with its sign removed, so $|x| \geqslant 0$ always —for example $|-5| = 5$ and $|5| = 5$.

The graph of $y = |ax + b|$ is a "**V**" shape that bounces off the x-axis.



Micro-check. Find the value of $|3 - 7|$. [1]

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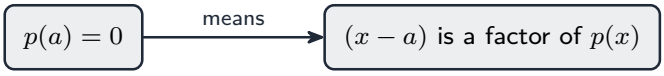
2 The factor and remainder theorems

A **polynomial** 多项式 is a sum of powers of x, such as $2x^3 + x - 5$; its **degree** 次数 is the highest power.

- **remainder theorem** 余数定理: the **remainder** 余数 when $p(x)$ is divided by $(x - a)$ is $p(a)$;
- **factor theorem** 因式定理: $(x - a)$ is a factor of $p(x)$ exactly when $p(a) = 0$.



Worked example. For $p(x) = x^3 - 2x^2 - 5x + 6$: $p(1) = 1 - 2 - 5 + 6 = 0$, so $(x - 1)$ **is** a factor.

Micro-check. For $p(x) = x^2 - 9$, is $(x - 3)$ a factor? Check $p(3)$. [1]

Watch out: the modulus $|x|$ is always $\geqslant 0$, and $|x| = k$ has *two* solutions ($x = k$ and $x = -k$). Factor theorem: $(x - a)$ is a factor when $p(a) = 0$; remainder theorem: the remainder on dividing by $(x - a)$ is $p(a)$.

Practice

2.1 Find (a) $|-8|$ and (b) $|2-11|$. [2]

2.2 Solve $|x| = 4$. [1]

2.3 The polynomial $p(x) = x^3 + 2x^2 - x - 2$. Find $p(1)$, and state whether $(x-1)$ is a factor. [2]

2.4 Use the remainder theorem to find the remainder when $p(x) = x^2 + 3x + 5$ is divided by $(x-2)$. [2]

2.5 State the degree of the polynomial $4x^3 - x^5 + 7$. [1]

2.6 Challenge. Show that $(x+2)$ is a factor of $p(x) = x^3 + 3x^2 - 4x - 12$, and hence state one value of x for which $p(x) = 0$. [3]

3 **(a)** Solve the inequality $|3x-4| \leqslant |2x+5|$. [4]

(b) Hence find the largest integer N satisfying the inequality $|3\times 7^{0.01N}-4| \leqslant |2\times 7^{0.01N}+5|$. [3]

2 Solve the equation $e^{2x}\left(e^{2x}-8\right)=48$. [3]

2 Solve the equation $2\tan^2\theta + 3\sec\theta = 18$ for $-180° < \theta < 180°$. [5]

2 Pure Mathematics 2 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)
logarithm	对数	
indices	指数	
laws of logarithms	对数定律	
natural logarithm	自然对数	

Recall

In **Part 1** you met the modulus and the factor and remainder theorems. Part 2 adds **logarithms** (the reverse of powers) and the **exponential function**.

New ideas

■ 1 Logarithms

A **logarithm** 对数 answers the question "**what power?**". If $a^x = y$ then $x = \log_a y$. Logarithms and **indices** 指数 (powers) are reverse ideas —for example $\log_2 8 = 3$ because $2^3 = 8$.

The **laws of logarithms** 对数定律 let you combine them:

$$\log(mn) = \log m + \log n, \quad \log\tfrac{m}{n} = \log m - \log n, \quad \log(m^k) = k \log m.$$

$$\log(mn) = \log m + \log n$$

$$\log \frac{m}{n} = \log m - \log n$$

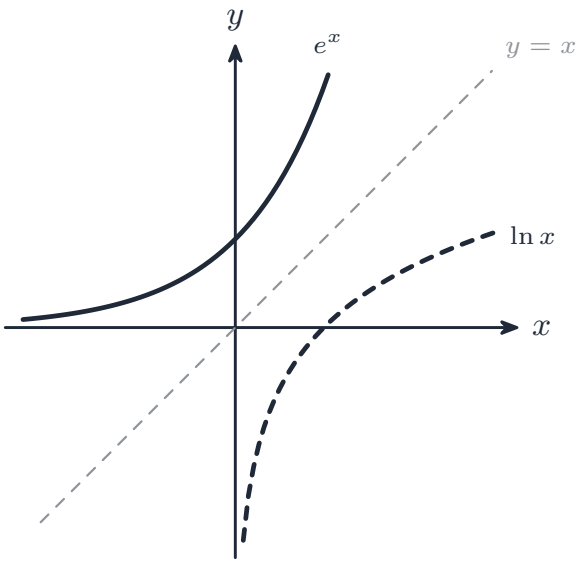
$$\log(m^k) = k \log m$$

Micro-check. Find $\log_3 9$. [1]

■ 2 The exponential function

The **exponential function** e^x and the **natural logarithm** 自然对数 $\ln x$ are **inverse** functions, so $\ln(e^x) = x$ and $e^{\ln x} = x$. Each is the other reflected in the line $y = x$.

When the unknown is **in the power**, take logs of both sides.



Worked example. Solve $2^x = 10$. Take logs: $x \ln 2 = \ln 10$, so $x = \dfrac{\ln 10}{\ln 2} \approx 3.32$.

Micro-check. Simplify $\ln(e^5)$. [1]

Watch out: a logarithm answers "what power?" —$\log_a y = x$ means $a^x = y$. Combine logs with the laws ($\log mn = \log m + \log n$, etc.). When the unknown is *in the power*, take logs of both sides. e^x and $\ln x$ are inverses.

Practice

2.1 Find (a) $\log_2 16$ and (b) $\log_{10} 1000$. [2]

2.2 Write $\log 6 + \log 5$ as a single logarithm. [1]

2.3 Write $\log(x^3)$ using the power law. [1]

2.4 Solve $3^x = 20$, giving x to 3 significant figures. [2]

2.5 Simplify (a) $\ln(e^7)$ and (b) $e^{\ln 4}$. [2]

2.6 Challenge. Solve $5^{x+1} = 100$, giving x to 3 significant figures. [3]

1 Given that $y = 6x\cos(x^2 + 1)$, find an expression for $\frac{dy}{dx}$. [2]

1 Find $\int 6\sin^2 x \, dx$. [3]

3 **(a)** Sketch, on a single diagram, the graphs of $y = 3e^{-2x}$ and $y = \sec x$ for values of x such that $0 \leqslant x < \frac{1}{2}\pi$. [2]

(b) Show that the x-coordinate of the point of intersection of the two graphs satisfies the equation $x = \frac{1}{2}\ln(3\cos x)$. [2]

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(c) Use an iterative formula, based on the equation in part **(b)**, to find the x-coordinate of the point of intersection correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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4 Mechanics 1 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
force	力			
resultant	合力			
equilibrium	平衡			
weight	重力			
friction	摩擦力			
normal reaction	法向反作用力			
coefficient of friction	摩擦系数			
Displacement	位移			
velocity	速度			
acceleration	加速度			
velocity-time graph	速度时间图			

Recall

At IGCSE you used speed, force and weight. Bring this with you:

- speed = distance ÷ time; a force is a push or a pull;
- weight pulls objects downward.

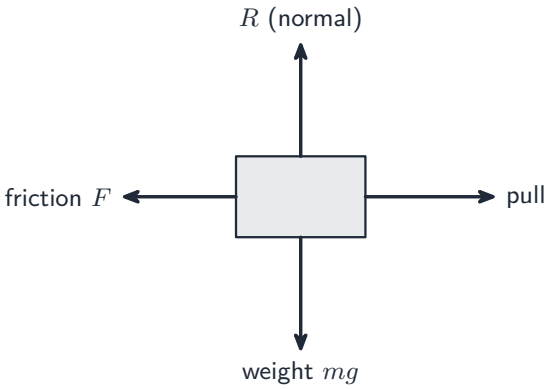
Mechanics 1 makes these exact, using forces as **vectors** and the constant-acceleration ("suvat") equations. (Take $g = 10$ m s^{-2}.)

New ideas

1 Forces and equilibrium

A **force** 力 is a push or a pull —a vector (it has size **and** direction). Several forces add to one **resultant** 合力.

A particle is in **equilibrium** 平衡 when the forces are **balanced** —the resultant is zero. Its **weight** 重力 is $W = mg$ (downward). On a rough surface, **friction** 摩擦力 opposes sliding, up to a maximum $F \leqslant \mu R$, where R is the **normal reaction** 法向反作用力 and μ the **coefficient of friction** 摩擦系数.



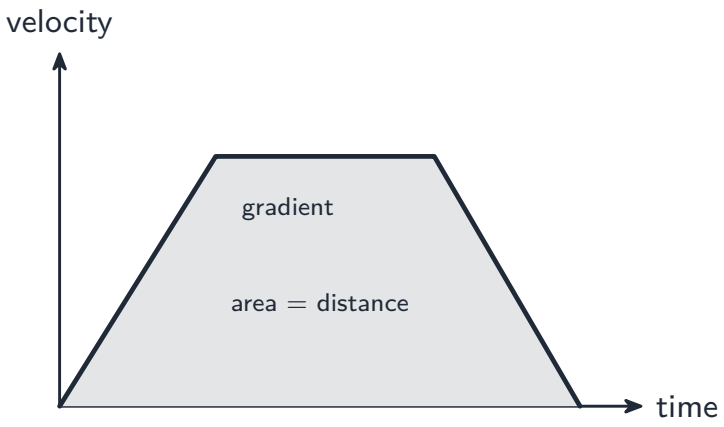
Micro-check. Find the weight of a 5 kg block (take $g = 10$ m s^{-2}). [1]

2 Kinematics

Displacement 位移, **velocity** 速度 and **acceleration** 加速度 are vectors. For **constant acceleration**, use the suvat equations:

$$v = u + at, \qquad s = ut + \tfrac{1}{2}at^2, \qquad v^2 = u^2 + 2as.$$

On a **velocity-time graph** 速度时间图 the **area** under the line is the distance, and the **gradient** is the acceleration.



Worked example. From rest ($u = 0$), $a = 2.5$ m s^{-2} for $t = 4$ s: $v = 0 + 2.5(4) = 10$ m s^{-1} and $s = \frac{1}{2}(2.5)(4^2) = 20$ m.

Micro-check. A car starts at $u = 3$ m s^{-1} and accelerates at $a = 2$ m s^{-2} for $t = 5$ s. Find v. [1]

Watch out: in equilibrium the resultant force is *zero* (the forces balance). For suvat, list u, v, a, s, t and pick the equation with only *one* unknown. On a velocity–time graph, area = distance and gradient = acceleration.

Practice

4.1 State what is true about the forces on a particle in **equilibrium**. [1]

4.2 Find the weight of a 12 kg mass ($g = 10$ m s^{-2}). [1]

4.3 A body starts from rest and accelerates at 3 m s^{-2} for 6 s. Find its final velocity. [2]

4.4 Using the same body, find the distance travelled in those 6 s. [2]

4.5 A block of weight 20 N rests on a rough surface with $\mu = 0.3$. The normal reaction is 20 N. Find the maximum friction force. [2]

4.6 Challenge. A ball is thrown straight up at 20 m s^{-1} ($g = 10$ m s^{-2}). Find the greatest height it reaches. [3]

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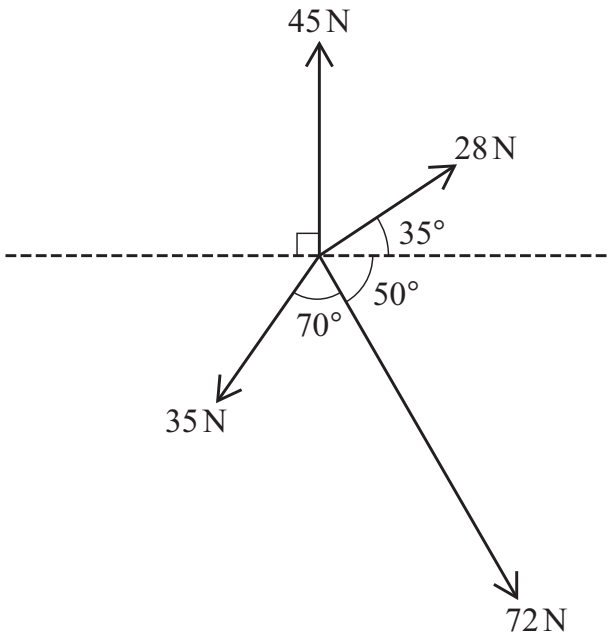

Coplanar forces of magnitudes 45 N, 28 N, 72 N and 35 N act at a point in the directions shown in the diagram.

Find, in either order, the magnitude and direction of the resultant force. [6]

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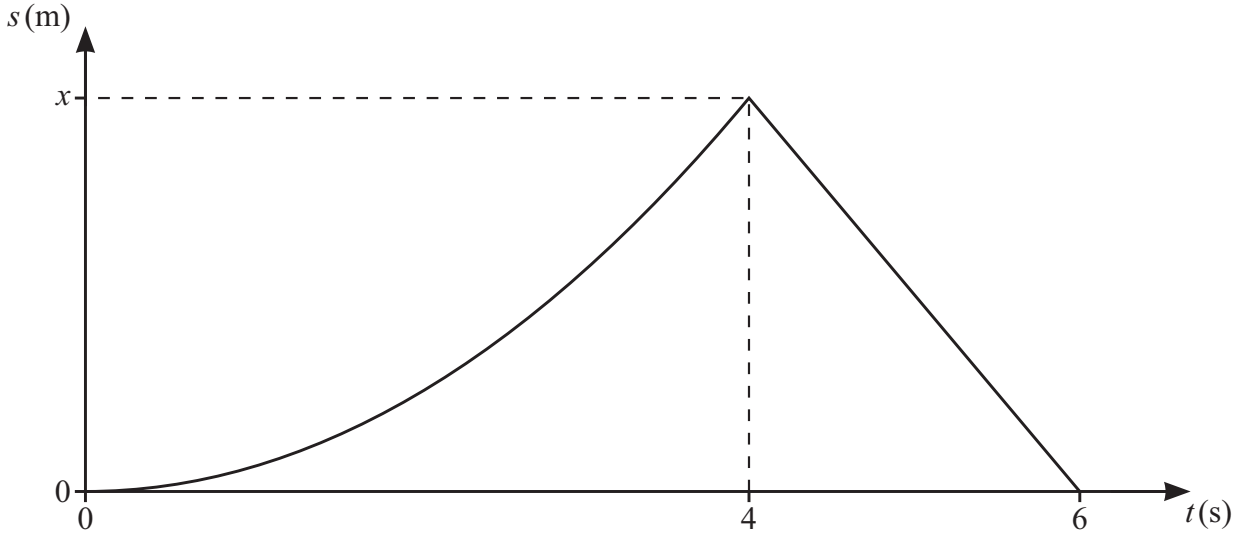


The diagram shows the displacement-time graph for the motion of a particle. The particle starts from rest at a point O and travels with constant acceleration $a\,\text{m s}^{-2}$, taking 4 s to move a distance of x m. The particle then returns to O with constant speed of $40\,\text{m s}^{-1}$, over a period of 2 s.

Find the value of x and the value of a. [3]

4 Mechanics 1 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
Newton's laws of motion	牛顿运动定律			
linear momentum	动量			
conservation of linear momentum	动量守恒			
work done	功			
kinetic energy	动能			
gravitational potential energy	重力势能			
power	功率			

Recall

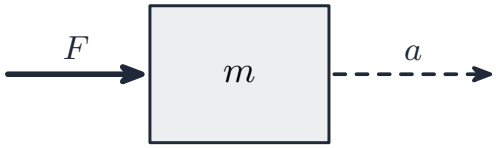
In **Part 1** you met forces, equilibrium, weight, friction, and the suvat equations. Part 2 adds **Newton's second law**, **momentum**, and **work, energy and power**.

New ideas

■ 1 Newton's second law

Newton's laws of motion 牛顿运动定律 connect force and acceleration. The key one is: resultant force = mass × acceleration,

$$F = ma.$$



$$F = ma$$

Worked example. A resultant force of 12 N acts on a 4 kg mass: $a = \dfrac{F}{m} = \dfrac{12}{4} = 3$ m s^{-2}.

Micro-check. A 6 N resultant force acts on a 2 kg mass. Find the acceleration. [1]

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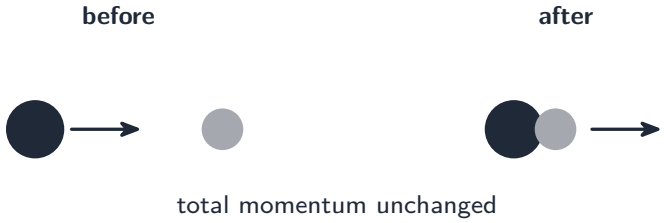
2 Momentum

The **linear momentum** 动量 of a body is mass × velocity (a vector). In a collision with no outside force, the total momentum is unchanged —the **conservation of linear momentum** 动量守恒:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$



Worked example. A 2 kg body at 3 m s^{-1} hits a stationary 1 kg body and they stick: $2(3) = 3v$, so $v = 2$ m s^{-1}.

Micro-check. Find the momentum of a 5 kg body moving at 4 m s^{-1}. [1]

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3 Work, energy and power

- **work done** 功 by a force is force × distance moved: $W = Fd$ (in joules);
- **kinetic energy** 动能 $= \frac{1}{2}mv^2$; **gravitational potential energy** 重力势能 $= mgh$;
- **power** 功率 is the rate of doing work; for a driving force, $P = Fv$.

Worked example. A car engine works at 12 kW while moving at 20 m s^{-1}: driving force $F = \dfrac{P}{v} = \dfrac{12000}{20} = 600$ N.

Micro-check. Find the kinetic energy of a 2 kg body moving at 3 m s^{-1}. [1]

Watch out: $F = ma$ uses the *resultant* force. In a collision with no outside force, *momentum* (mv) is conserved: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$. Power for a driving force is $P = Fv$.

Practice

4.1 A resultant force of 20 N acts on a 5 kg mass. Find the acceleration. [2]

4.2 Find the momentum of a 1500 kg car moving at 12 m s^{-1}. [2]

4.3 A 3 kg body at 4 m s^{-1} hits a stationary 1 kg body and they stick together. Find their common speed. [2]

4.4 Find the kinetic energy of a 4 kg body moving at 5 m s^{-1}. [2]

4.5 A force pulls a load at 8 m s^{-1} with a driving force of 50 N. Find the power ($P =$

Fv). [2]

4.6 Challenge. A 2 kg trolley moving at 5 m s^{-1} collides with a stationary 3 kg trolley and they stick together. Find their common speed, and the kinetic energy lost in the collision. [3]

5 When a particle P of mass m kg has speed u m s^{-1}, its momentum is 4 N s and its kinetic energy is 16 J.

(a) Find the value of m and the value of u. [3]

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P is now projected on a smooth horizontal surface with speed v m s^{-1} directly towards a particle Q of mass 1.25 kg which is stationary. After P and Q collide, the velocity of P is w m s^{-1} and the velocity of Q is $2w$ m s^{-1}. The loss of kinetic energy in the collision is 25 J.

(b) Find the value of v and the value of w. [4]

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1 A block of mass 12 kg is being pulled by a rope up a rough plane. The plane is inclined at an angle of 20° above the horizontal. The rope pulling the block is parallel to a line of greatest slope of the plane. The coefficient of friction between the block and the plane is 0.4. The acceleration of the block is $2\,\text{m}\,\text{s}^{-2}$.

Find the tension in the rope. [4]

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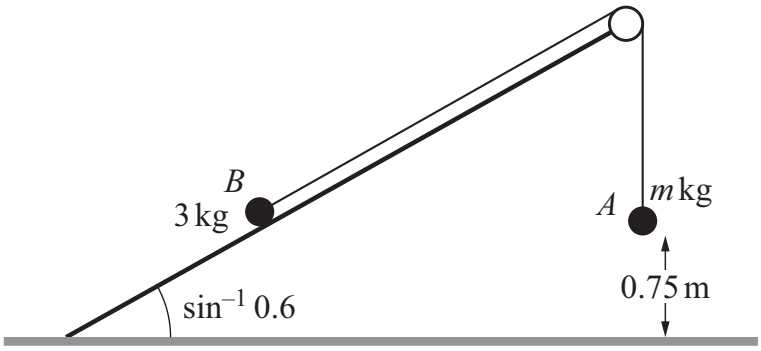


Two particles, A and B, of masses m kg and 3 kg respectively, are connected by a light inextensible string. Particle B is on a fixed plane which is at an angle of $\sin^{-1} 0.6$ to the horizontal ground. The string passes over a fixed smooth pulley at the top of the plane. Particle A hangs vertically below the pulley and is 0.75 m above the ground (see diagram).

The system is released from rest. In the subsequent motion B moves up a line of greatest slope of the plane and does not reach the pulley. As B moves up the plane there is a constant resistance to its motion of magnitude $\frac{10}{3}$ N. The speed of A immediately before it hits the ground is $2\,\text{m}\,\text{s}^{-1}$.

Use an energy method to find the value of m. [6]

5 Probability & Statistics 1 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
mean	平均数			
median	中位数			
mode	众数			
range	极差			
standard deviation	标准差			
permutation	排列			
combination	组合			

Recall

At IGCSE you summarised data and found simple probabilities. Bring this with you:

- the **mean** 平均数, **median** 中位数 and **mode** 众数 of a set of numbers;
- listing outcomes to find a probability.

Statistics 1 adds a measure of **spread**, and a way to **count** arrangements.

New ideas

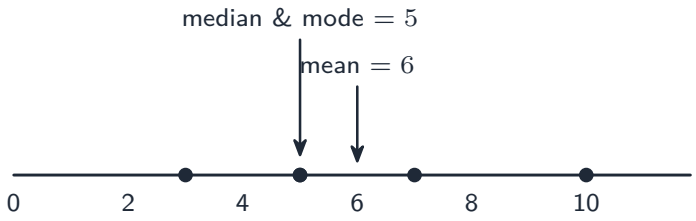
1 Averages and spread

A "middle" value (a measure of central tendency):

- the **mean** 平均数 $\bar{x} = \dfrac{\sum x}{n}$; the **median** 中位数 (the middle value in order); the **mode** 众数 (the most common value).

A measure of spread says how spread out the data is:

- the **range** 极差 = highest − lowest;
- the **standard deviation** 标准差 $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$.



Worked example. For 10 values with $\sum x = 50$ and $\sum x^2 = 300$: $\bar{x} = \frac{50}{10} = 5$ and $\sigma = \sqrt{\frac{300}{10} - 5^2} = \sqrt{5} = 2.24$.

Micro-check. Find the **mean** of $4, 6, 8, 10, 12$. [1]

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2 Permutations and combinations

- a **permutation** 排列 is an arrangement where **order matters** (AB is different from BA);
- a **combination** 组合 is a selection where **order does not matter** ($\{A, B\}$ is the same as $\{B, A\}$).

The counts are ${}^nP_r = \dfrac{n!}{(n-r)!}$ and ${}^nC_r = \dfrac{n!}{r!\,(n-r)!}$.

Micro-check. When choosing a team where order does **not** matter, do you use a permutation or a combination? [1]

Watch out: $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ —square the mean *before* subtracting. Permutation (order *matters*) uses ${}^nP_r$; combination (order does *not*) uses ${}^nC_r$, which is always smaller.

Practice

5.1 Find the **mean**, **median** and **mode** of $3, 5, 5, 7, 10$. [3]

5.2 Find the **range** of $12, 4, 9, 20, 7$. [1]

5.3 For 8 values, $\sum x = 40$ and $\sum x^2 = 218$. Find the mean and the standard deviation. [3]

5.4 State whether **order matters** for (a) a permutation, (b) a combination. [2]

5.5 Find ${}^5C_2 = \dfrac{5!}{2!\,3!}$. [2]

5.6 Challenge. A committee of 3 is chosen from 7 people. How many different commit-

tees are possible? [2]

3 The back-to-back stem-and-leaf diagram shows the annual salaries, in dollars, of 27 employees at each of two companies, Browns and Greens.

Browns									Greens								
(3)					9	8	4	30	6	8							(2)
(7)	8	8	5	3	3	1	0	31	2	4	5	8					(4)
(7)	9	7	6	4	2	2	0	32	3	6	6	7	9				(5)
(6)		8	7	5	5	3	1	33	1	1	3	7	8	8	9	9	(8)
(3)					4	2	2	34	0	2	3	5	6	9			(6)
(1)							7	35	3	7							(2)

Key: 6|32|7 means \$32 600 for Browns and \$32 700 for Greens.

(a) Find the median and interquartile range for the annual salaries of employees at Browns. [3]

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The annual salary of an employee at Browns is denoted by x thousand dollars and the annual salary of an employee at Greens is denoted by y thousand dollars. It is given that, for the 27 employees at each of the companies,

$$\Sigma x = 878.3, \quad \Sigma x^2 = 28\,616.09, \quad \Sigma y = 896.5, \quad \Sigma y^2 = 29\,815.63.$$

(b) Find the mean and standard deviation of the annual salaries of these 54 employees. [5]

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2 **(a)** Find the number of different arrangements of the 8 letters in the word KANGAROO in which the two As are together and the two Os are **not** together. [3]

A fair 8-sided dice has faces labelled K, A, N, G, A, R, O, O. The dice is rolled repeatedly.

(b) Find the probability that fewer than 6 rolls of this dice are required to obtain an A. [2]

(c) Find the probability that the second A is obtained on the 6th roll of the dice. [2]

5 Probability & Statistics 1 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
probability	概率			
independent events	独立事件			
mutually exclusive events	互斥事件			
discrete random variable	离散型随机变量			
expectation	期望			
binomial distribution	二项分布			

Recall

In **Part 1** you met averages, spread, and how to count arrangements (permutations and combinations). Part 2 looks at **combining probabilities**, and at **random variables**.

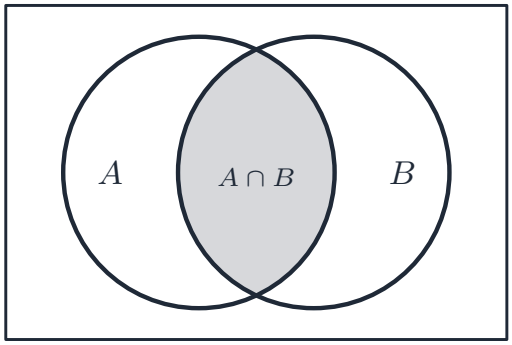
New ideas

■ 1 Probability rules

Find a **probability** 概率 by counting equally likely outcomes. Combine probabilities with these rules:

- **addition** (for "or"): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
- **multiplication** (for "and", when events are **independent events** 独立事件): $P(A \cap B) = P(A)\,P(B)$.

Two events are **mutually exclusive events** 互斥事件 if they cannot both happen (then $P(A \cap B) = 0$).



Worked example. $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Since $0.5 \times 0.4 = 0.2 = P(A \cap B)$, the events are **independent**.

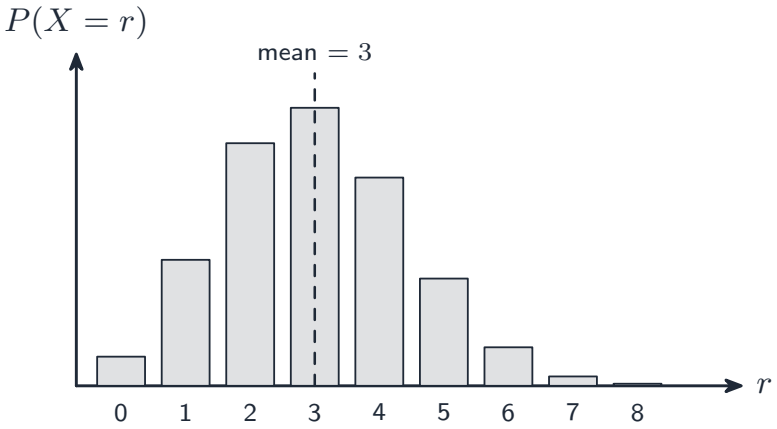
Micro-check. A fair die is rolled. Find the probability of getting a 5. [1]

■ 2 Discrete random variables

A **discrete random variable** 离散型随机变量 X takes separate values, each with a probability (the probabilities add to 1). Its **expectation** 期望 (mean) is

$$E(X) = \sum x\, P(X = x).$$

The **binomial distribution** 二项分布 $X \sim B(n, p)$ counts the successes in n independent trials, with $E(X) = np$.



Worked example. For $X \sim B(10, 0.3)$: $E(X) = np = 10 \times 0.3 = 3$.

Micro-check. For $X \sim B(20, 0.5)$, find $E(X)$. [1]

> **Watch out:** for "or" use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; for independent "and" use $P(A \cap B) = P(A)P(B)$. Test independence by checking whether $P(A)P(B) = P(A \cap B)$. For $X \sim B(n, p)$, $E(X) = np$.

Practice

5.1 A fair die is rolled. Find the probability of getting an **even** number. [1]

5.2 $P(A) = 0.3$, $P(B) = 0.5$, $P(A \cap B) = 0.1$. Find $P(A \cup B)$. [2]

5.3 $P(A) = 0.4$, $P(B) = 0.25$, and $P(A \cap B) = 0.1$. Test whether A and B are **independent**. [2]

5.4 A random variable X has $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, $P(X = 3) = 0.3$. Find $E(X)$. [2]

5.5 For $X \sim B(40, 0.25)$, find $E(X)$. [1]

5.6 Challenge. A fair coin is tossed 6 times. Using the binomial distribution, find the

expected number of heads and the probability of getting exactly 6 heads. [3]

2 A fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6 is thrown repeatedly until a 6 is obtained.

(a) Find the probability that a 6 is obtained for the first time on the 8th throw. [1]

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(b) Find the probability that a 6 is obtained for the third time on the 7th throw. [3]

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1 The random variable X takes the value x with probability kx^2, where k is a constant and x takes the values $-2, 1, 2, 3$ only.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

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(b) Find $\mathrm{E}(X)$. [1]

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(c) Find $\mathrm{P}(X \neq 2 \,|\, X > 0)$. [2]

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1 The masses of the bags of rice made by a company are normally distributed with mean μkg and standard deviation 0.14 kg. The probability that the mass of a randomly chosen bag of this rice is less than 1.48 kg is 0.22.

Find the value of μ. [3]