

2	$\uparrow N_1 \cos \theta_1 = mg$ $\rightarrow N_1 \sin \theta_1 = m(r \sin \theta_1) \omega_1^2$
	$\uparrow N_2 \cos \theta_2 = mg$ $\rightarrow N_2 \sin \theta_2 = m(r \sin \theta_2) \omega_2^2$
	$\frac{\frac{5}{4}g}{\frac{5}{3}g} = \frac{r\omega_1^2}{r\omega_2^2}$
	$\frac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$

3(a)	$\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+A]$
	$t = 0, v = 2 : A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$
	$v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+B]$
	$x = 0, t = 0 : B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \quad \left[= \ln\left(2\cos^2\left(t - \frac{1}{4}\pi\right)\right)\right]$
3(b)	$2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$
	$t = \frac{1}{2}\pi$

4

In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the length of the string when speed of P is $\sqrt{\frac{7}{5}ag}$.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 60ax + 27a^2 = 0$$

$$x = \frac{9}{5}a \text{ only.}$$

Alternative method for question 4

In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the extension when P is released.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mgx^2}{2a} = mg(e-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 10ax - 8a^2 = 0$$

$$(5x+2a)(5x-4a) = 0, \quad x = \frac{4}{5}a \text{ only.}$$

Required length of string = $\frac{9}{5}a$.

5(a)

$$\bar{x} = ka + \frac{1}{3}h$$

$$\bar{x} = \frac{1}{3}(3ka + h)$$

$$\bar{y} = a + \frac{1}{3}(h-a) = \frac{1}{3}(2a + h)$$

Alternative method for question 5(a)

Coordinates $A(ka, 0)$, $B(ka+h, h)$, $C(ka, 2a)$

$$\bar{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$$

$$\bar{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$$

5(b)	Taking moments about the x -axis: $\bar{x}(2ka^2 + ah) = 2ka^2 \times \frac{1}{2}ka + ah(k a + \frac{1}{3}h)$
	For equilibrium, $\bar{x} \leq ka$, so $\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$
	$(0 \leq) h \leq \sqrt{3}ka$
5(c)	Point of toppling means $h = \sqrt{3}ka [= a]$. $\bar{x} = \frac{1}{3}a(\sqrt{3} + 1)$
	[$h = a$, meaning triangle ABC is isosceles.] $\bar{y} = a$ (by symmetry)

7(a)	$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$
	$\frac{dy}{dx} = -1$
	$-1 = \frac{4}{3} - \frac{2}{45}x_A$
	$x_A = \frac{105}{2}$
	$y_A = \frac{35}{4}$
Alternative method for question 7(a)	
Horizontally: $v_H = 25 \cos \theta = 15$ Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$	
At A : $\frac{v_V}{v_H} = -\tan 45^\circ = -1$	
$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$	
$x_A = \frac{105}{2}$	
$y_A = \frac{35}{4}$	

7(b)	Let v be the velocity before the collision at A . $u_x = u \cos \theta = 15 = v_x$
	$v_y = -v_x = -15$
	Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (ms}^{-1}\text{)}$
7(c)	Let w be the velocity after the collision. $w = ev = \frac{5}{3}\sqrt{2}$
	$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$ OR $\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, t = \frac{3}{2}$ $x = (w \cos 45)t \left[= \frac{5}{3} \times \frac{3}{2} \right]$
	$x = \frac{5}{2} \text{ (metres)}$