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$$\uparrow N_1 \cos \theta_1 = mg$$

$$\rightarrow N_1 \sin \theta_1 = m(r \sin \theta_1) \omega_1^2$$

$$\uparrow N_2 \cos \theta_2 = mg$$

$$\rightarrow N_2 \sin \theta_2 = m(r \sin \theta_2) \omega_2^2$$

$$\frac{\frac{5}{4}g}{\frac{5}{3}g} = \frac{r\omega_1^2}{r\omega_2^2}$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$$

3(a)	$\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+A]$
	$t = 0, v = 2 : \quad A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$
	$v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+B]$
	$x = 0, t = 0 : \quad B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \quad \left[= \ln\left(2\cos^2\left(t - \frac{1}{4}\pi\right)\right)\right]$
3(b)	$2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$
	$t = \frac{1}{2}\pi$

4

In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the length of the string when speed of P is $\sqrt{\frac{7}{5}ag}$.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 60ax + 27a^2 = 0$$

$$x = \frac{9}{5}a \text{ only.}$$

Alternative method for question 4

In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the extension when P is released.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mgx^2}{2a} = mg(e-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 10ax - 8a^2 = 0$$

$$(5x+2a)(5x-4a) = 0, \quad x = \frac{4}{5}a \text{ only.}$$

Required length of string = $\frac{9}{5}a$.

5(a)

$$\bar{x} = ka + \frac{1}{3}h$$

$$\bar{x} = \frac{1}{3}(3ka + h)$$

$$\bar{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$$

Alternative method for question 5(a)

Coordinates $A(ka, 0)$, $B(ka + h, h)$, $C(ka, 2a)$

$$\bar{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$$

$$\bar{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$$

5(b)

Taking moments about the x -axis:

$$\bar{x}(2ka^2 + ah) = 2ka^2 \times \frac{1}{2}ka + ah\left(ka + \frac{1}{3}h\right)$$

For equilibrium, $\bar{x} \leq ka$, so

$$\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$$

$$(0 \leq) h \leq \sqrt{3}ka$$

5(c)

Point of toppling means $h = \sqrt{3}ka [= a]$.

$$\bar{x} = \frac{1}{3}a(\sqrt{3} + 1)$$

[$h = a$, meaning triangle ABC is isosceles.] $\bar{y} = a$ (by symmetry)

7(a)

$$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$$

$$\frac{dy}{dx} = -1$$

$$-1 = \frac{4}{3} - \frac{2}{45}x_A$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

Alternative method for question 7(a)

Horizontally: $v_H = 25 \cos \theta = 15$

Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$

At A: $\frac{v_V}{v_H} = -\tan 45^\circ = -1$

$$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

7(b)

Let v be the velocity before the collision at A .

$$u_x = u \cos \theta = 15 = v_x$$

$$v_y = -v_x = -15$$

$$\text{Speed: } \sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (ms}^{-1}\text{)}$$

7(c)

Let w be the velocity after the collision.

$$w = ev = \frac{5}{3}\sqrt{2}$$

$$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \quad \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$$

OR

$$\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, \quad t = \frac{3}{2}$$

$$x = (w \cos 45)t \quad \left[= \frac{5}{3} \times \frac{3}{2} \right]$$

$$x = \frac{5}{2} \text{ (metres)}$$