2 A particle P of mass m is moving in a horizontal circle with angular speed ω_1 on the smooth inner surface of a hemispherical shell of radius r. The angle between the upward vertical and the normal reaction of the surface on P is θ_1, where $\tan\theta_1 = \frac{3}{4}$.

When the angular speed is increased to ω_2, the angle between the upward vertical and the normal reaction of the surface on P becomes θ_2, where $\tan\theta_2 = \frac{4}{3}$.

Find the ratio $\dfrac{\omega_1}{\omega_2}$. [4]

3 A particle P is moving in a straight horizontal line. At time t s, the displacement of P from a fixed point O on the line is x m and the velocity of P is $v\,\mathrm{m\,s^{-1}}$. The acceleration of P is $\frac{1}{2}(v^2+4)\,\mathrm{m\,s^{-2}}$ in the direction PO. Initially P is at O and is moving with velocity $2\,\mathrm{m\,s^{-1}}$.

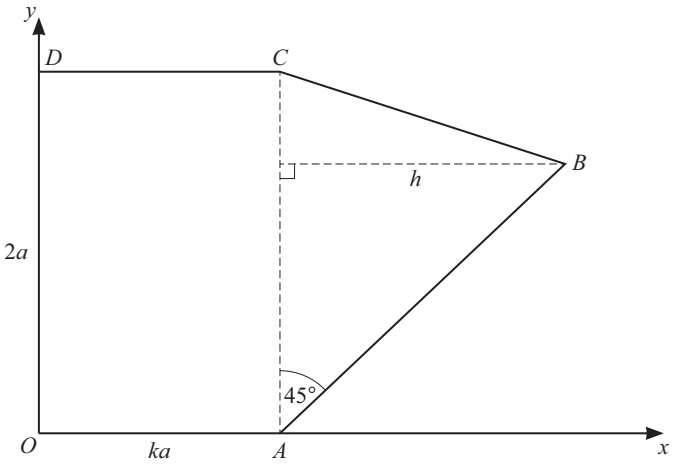
(a) Find an expression for x in terms of t. [5]

(b) Find the time when P next goes through O. [2]

4 One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O. Two particles, P and Q, of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

Find, in terms of a, the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

5



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC. The length of OA is ka, the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O, parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\overline{y}$. [3]

The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

(b) Find, in terms of a and k, the set of values of h for which the lamina is in equilibrium. [4]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

(c) Find, in terms of a, the coordinates of the centre of mass of the triangle ABC. [2]

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $25\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A, the direction of motion of P makes an angle of $45°$ with the downward vertical through A.

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A. [5]

At point A, the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

(b) Find the speed of P immediately before it collides with the barrier. [3]

(c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.