

1	$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = \begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7$ $k = \pm\sqrt{7}$
2(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1 + \sinh t}{\cosh t}$
2(b)	$\frac{d}{dt} \left(\frac{1 + \sinh t}{\cosh t} \right) = \frac{\cosh t (\cosh t) - (1 + \sinh t) \sinh t}{\cosh^2 t} = \frac{1 - \sinh t}{\cosh^2 t}$ $\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx} = \frac{1 - \sinh t}{\cosh^3 t}$
2(c)	$y = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) = 1 + x + \frac{1}{2} x^2$
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$

3(a)	$\frac{d}{dx} \left(x(1+x^2)^n \right) = 2nx^2(1+x^2)^{n-1} + (1+x^2)^n$ $= 2n(1+x^2-1)(1+x^2)^{n-1} + (1+x^2)^n$ $\left[x(1+x^2)^n \right]_0^1 = 2nI_n - 2nI_{n-1} + I_n$ $2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$
3(b)	$I_{-1} = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4} \pi$ $-I_{-1} = \frac{1}{2} - 2I_{-2}$ $-\frac{1}{4} \pi = \frac{1}{2} - 2I_{-2} \Rightarrow I_{-2} = \frac{1}{4} + \frac{1}{8} \pi$

4	$5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}i$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right)$
	$y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$
	$p = 1 \quad 4p + q = 5 \quad 10p + 2q + r = 3$
	$q = 1 \quad r = -9$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + x^2 + x - 9$
	$y' = e^{-\frac{1}{5}x} \left(-\frac{2}{5}A \sin \frac{2}{5}x + \frac{2}{5}B \cos \frac{2}{5}x \right) - \frac{1}{5}e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + 2x + 1$
	$A - 9 = 0 \quad \frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A = 9, B = 2$
	$y = e^{-\frac{1}{5}x} \left(9 \cos \frac{2}{5}x + 2 \sin \frac{2}{5}x \right) + x^2 + x - 9$

$\frac{4}{3n} \rightarrow 0$ as $n \rightarrow \infty$

7(a)	$\frac{d}{dx} \left(\tanh^{-1} x \right) = \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$
	$= \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{2}{(1-x)^2} = \frac{1}{1-x^2}$
	Alternative method for question 7(a)
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = 1$
	$\frac{dy}{dx} = \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$
7(b)	$\frac{dy}{dx} - \frac{y}{x} = x \tanh^{-1} x$
	$e^{-\int x^{-1} d\theta} = e^{-\ln x} = x^{-1}$
	$\frac{d}{dx} (x^{-1} y) = \tanh^{-1} x$
	$x^{-1} y = x \tanh^{-1} x - \int \frac{x}{1-x^2} dx$
	$x^{-1} y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + C$
	$0 = \frac{1}{2} \tanh^{-1} \frac{1}{2} + \frac{1}{2} \ln \frac{3}{4} + C \Rightarrow C = -\frac{1}{4} \ln 3 - \frac{1}{2} \ln \frac{3}{4}$
	$x^{-1} y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16}$