1	$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = \begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7$
	$k = \pm\sqrt{7}$

2(a)	$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{1+\sinh t}{\cosh t}$
2(b)	$\dfrac{d}{dt}\left(\dfrac{1+\sinh t}{\cosh t}\right) = \dfrac{\cosh t(\cosh t) - (1+\sinh t)\sinh t}{\cosh^2 t} = \dfrac{1-\sinh t}{\cosh^2 t}$
	$\dfrac{d^2 y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx} = \dfrac{1-\sinh t}{\cosh^3 t}$
2(c)	$y = y(0) + xy'(0) + \dfrac{x^2}{2!} y''(0) = 1 + x + \tfrac{1}{2}x^2$
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$

3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x\left(1+x^2\right)^n\right)=2nx^2\left(1+x^2\right)^{n-1}+\left(1+x^2\right)^n$
	$=2n\left(1+x^2-1\right)\left(1+x^2\right)^{n-1}+\left(1+x^2\right)^n$
	$\left[x\left(1+x^2\right)^n\right]_0^1=2nI_n-2nI_{n-1}+I_n$
	$2^n=(2n+1)I_n-2nI_{n-1}\Rightarrow(2n+1)I_n=2^n+2nI_{n-1}$
3(b)	$I_{-1}=\left[\tan^{-1}x\right]_0^1=\frac{1}{4}\pi$
	$-I_{-1}=\frac{1}{2}-2I_{-2}$
	$-\frac{1}{4}\pi=\frac{1}{2}-2I_{-2}\Rightarrow I_{-2}=\frac{1}{4}+\frac{1}{8}\pi$

4	$5m^2+2m+1=0 \Rightarrow m=-\frac{1}{5}\pm\frac{2}{5}\mathrm{i}$
	$y=\mathrm{e}^{-\frac{1}{5}x}\left(A\cos\frac{2}{5}x+B\sin\frac{2}{5}x\right)$
	$y=px^2+qx+r \Rightarrow y'=2px+q \Rightarrow y''=2p$
	$p=1 \quad 4p+q=5 \quad 10p+2q+r=3$
	$q=1 \quad r=-9$
	$y=\mathrm{e}^{-\frac{1}{5}x}\left(A\cos\frac{2}{5}x+B\sin\frac{2}{5}x\right)+x^2+x-9$
	$y'=\mathrm{e}^{-\frac{1}{5}x}\left(-\frac{2}{5}A\sin\frac{2}{5}x+\frac{2}{5}B\cos\frac{2}{5}x\right)-\frac{1}{5}\mathrm{e}^{-\frac{1}{5}x}\left(A\cos\frac{2}{5}x+B\sin\frac{2}{5}x\right)+2x+1$
	$A-9=0 \quad \frac{2}{5}B-\frac{1}{5}A+1=0 \Rightarrow A=9, B=2$
	$y=\mathrm{e}^{-\frac{1}{5}x}\left(9\cos\frac{2}{5}x+2\sin\frac{2}{5}x\right)+x^2+x-9$

	$\frac{4}{3n}\to 0$ as $n\to\infty$

7(a)	$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$
	$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{2}{(1-x)^2} = \frac{1}{1-x^2}$
	Alternative method for question 7(a)
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = 1$
	$\frac{dy}{dx} = \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2}$
7(b)	$\frac{dy}{dx} - \frac{y}{x} = x\tanh^{-1} x$
	$e^{-\int x^{-1}\,d\theta} = e^{-\ln x} = x^{-1}$
	$\frac{d}{dx}\left(x^{-1}y\right) = \tanh^{-1} x$
	$x^{-1}y = x\tanh^{-1} x - \int \frac{x}{1-x^2}\,dx$
	$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) + C$
	$0 = \frac{1}{2}\tanh^{-1}\frac{1}{2} + \frac{1}{2}\ln\frac{3}{4} + C \Rightarrow C = -\frac{1}{4}\ln 3 - \frac{1}{2}\ln\frac{3}{4}$
	$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) - \frac{1}{4}\ln\frac{27}{16}$