

Further Mathematics: 9231 Mathematics - Further November 2025 Question Paper 21

1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t .

[2]

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$.

[4]

(c) Find the Maclaurin's series for y in terms of x up to and including the term in x^2 . [2]

3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n dx$.

(a) By considering $\frac{d}{dx}(x(1+x^2)^n)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}. \quad [5]$$

[4]

⑤

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form.

[9]

⑥

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.