

1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t . [2]

This image shows a single sheet of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the width of the page. There is no text or other markings on the paper.

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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- (c)** Find the Maclaurin's series for y in terms of x up to and including the term in x^2 . [2]

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(b)

 $I_{-2}.$

[4]

5

7 (a) Show that $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$.

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

