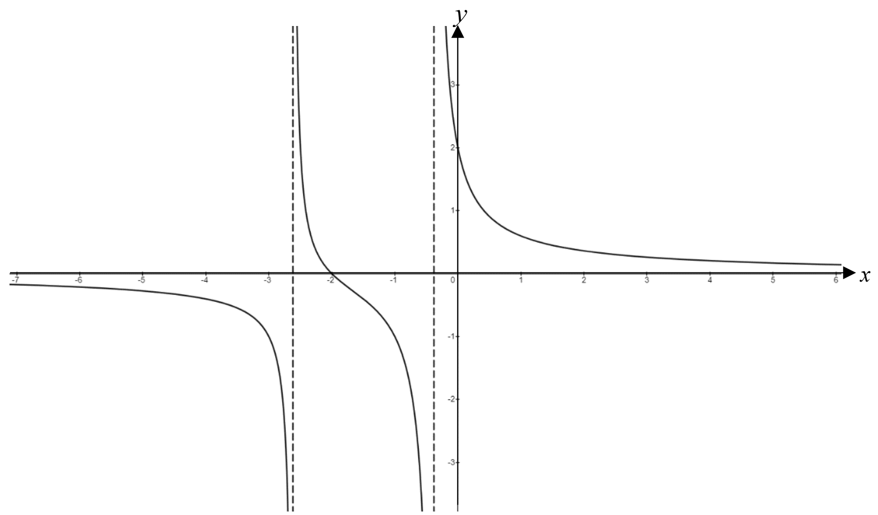
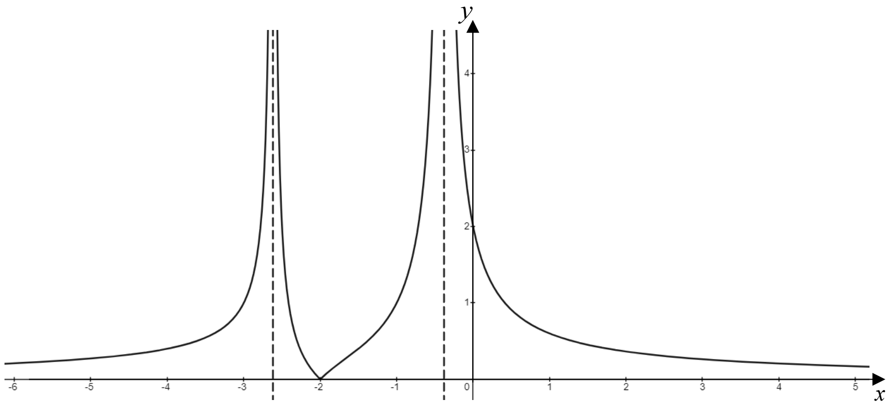
4(a)	$y=2x+1 \Rightarrow x=\frac{1}{2}(y-1)$
	$\frac{1}{16}(y-1)^4+\frac{1}{8}(y-1)^3+\frac{1}{4}(y-1)^2+\frac{1}{2}(y-1)+1=0$ $\frac{1}{16}\left(y^4-4y^3+6y^2-4y+1\right)+\frac{1}{8}\left(y^3-3y^2+3y-1\right)+\frac{1}{4}\left(y^2-2y+1\right)+\frac{1}{2}(y-1)+1=0$
	$\frac{1}{16}y^4-\frac{1}{8}y^3+\frac{1}{4}y^2+\frac{1}{8}y+\frac{11}{16}=0 \Rightarrow y^4-2y^3+4y^2+2y+11=0$
4(b)	$S_2=2^2-2(4)$
	$S_2=-4$
4(c)	$S_4-2S_3+4S_2+2S_1+44=0$
	$S_4=-76$

7(a)	$x=-\frac{3}{2}-\frac{1}{2}\sqrt{5}, \quad x=-\frac{3}{2}+\frac{1}{2}\sqrt{5}$
	$y=0$
7(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+3x+1)(1)-(x+2)(2x+3)}{\left(x^2+3x+1\right)^2}$
	$x^2+4x+5=0$
	Discriminant is $4^2-20=-4<0$ so no real roots.

7(c)	

7(d)	
7(e)	$\frac{x+2}{x^2+3x+1}=2$ or $\frac{x+2}{x^2+3x+1}=-2$ $-2x^2-5x=0$ or $2x^2+7x+4=0$
	$x=-\frac{5}{2},\quad x=0$ or $x=-\frac{7}{4}-\frac{1}{4}\sqrt{17},\quad x=-\frac{7}{4}+\frac{1}{4}\sqrt{17}$
	$-\frac{7}{4}-\frac{1}{4}\sqrt{17}<x<-\frac{5}{2},\quad -\frac{7}{4}+\frac{1}{4}\sqrt{17}<x<0\ ,\ x\neq-\frac{3}{2}\pm\frac{1}{2}\sqrt{5}$

5(a)(i)	$\cos\left(\frac{1}{4}\pi-\frac{1}{6}\pi\right)=\cos\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right)+\sin\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right)$ $=\frac{1}{4}\sqrt{2}\sqrt{3}+\frac{1}{4}\sqrt{2}=\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$
5(a)(ii)	$\sin\left(\frac{1}{4}\pi-\frac{1}{6}\pi\right)=\sin\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right)-\cos\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right)$ $=\frac{1}{4}\sqrt{2}\sqrt{3}-\frac{1}{4}\sqrt{2}=\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$
5(b)	Reflection and rotation.
	Correct order.
	Reflection in the x-axis.
	A rotation, $\frac{1}{12}\pi$ [anticlockwise] about the origin.

5(c)	$\begin{pmatrix} \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) & \frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right) \\ \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right) & \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) & \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right) \\ \frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right) & \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
	$\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) & \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right) \\ \frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right) & \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right) \end{pmatrix}$

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5(d)	$\mathbf{M} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix}$
	$\begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right)x+\sin\left(\frac{\pi}{12}\right)y \\ \sin\left(\frac{\pi}{12}\right)x-\cos\left(\frac{\pi}{12}\right)y \end{pmatrix}$
	$\sin\left(\frac{\pi}{12}\right)x - m\cos\left(\frac{\pi}{12}\right)x = m\left(\cos\left(\frac{\pi}{12}\right)x + \sin\left(\frac{\pi}{12}\right)mx\right)$
	$\sin\left(\frac{\pi}{12}\right)m^2 + 2m\cos\left(\frac{\pi}{12}\right) - \sin\left(\frac{\pi}{12}\right) = 0$
	$m = \dfrac{-2\cos\left(\frac{\pi}{12}\right) \pm \sqrt{4\cos^2\left(\frac{\pi}{12}\right)+4\sin^2\left(\frac{\pi}{12}\right)}}{2\sin\left(\frac{\pi}{12}\right)}$
	$m = -\cot\left(\frac{\pi}{12}\right) \pm \operatorname{cosec}\left(\frac{\pi}{12}\right)$