

Further Probability & Statistics

A-Level Further Mathematics ■ Topic 4

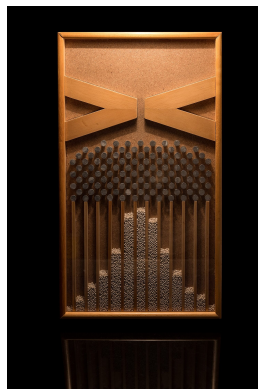
A-Level Further Mathematics



Further Probability & Statistics

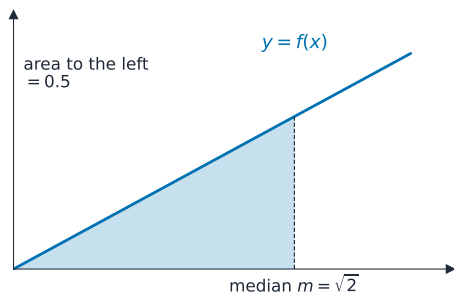
What we will cover

- 1 Continuous random variables
- 2 t -distribution inference
- 3 Chi-squared tests
- 4 Non-parametric tests
- 5 Generating functions
- 6 Recap



1 Continuous and inference

Continuous random variables



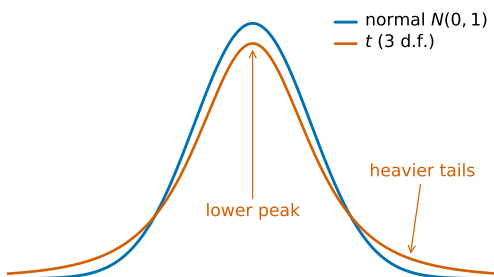
median: area 0.5 to its left

$$F(x) = P(X \leq x) = \int_{-\infty}^x f, \text{ and } f = F'.$$

$$f(x) = \frac{1}{2}x, 0 \leq x \leq 2$$

$$F = \frac{1}{4}x^2; \text{ median } m = \sqrt{2} = 1.41$$

t-distribution inference



Small sample, unknown variance $\Rightarrow$
use the **t-distribution** t 分布:

$$\bar{x} \pm t \frac{s}{\sqrt{n}} \quad (n - 1 \text{ d.f.}).$$

$$n = 10, \bar{x} = 50, s = 4, t = 2.262$$

$$50 \pm 2.86 \Rightarrow (47.1, 52.9)$$

2

Tests

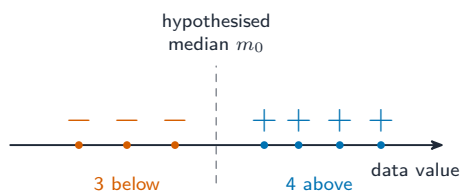
Chi-squared tests

Compare observed counts O with expected E :

$$\chi^2 = \sum \frac{(O - E)^2}{E}.$$

Large $\chi^2 \Rightarrow$ reject H_0 (the model does not fit). Degrees of freedom = categories $- 1$, minus any estimated parameters.

Non-parametric tests



No assumption that the data is normal:

- **sign test** 符号检验 – counts above/below a median
- **Wilcoxon** 威尔科克森 signed-rank & rank-sum – use the sizes too

Probability generating functions

$$G(t) = E(t^X) = \sum_x P(X=x) t^x.$$

$E(X) = G'(1)$; the PGF of a sum of independent variables is the **product** of their PGFs.

$$P = 0.5, 0.3, 0.2 \text{ for } 0, 1, 2$$

$$G = 0.5 + 0.3t + 0.2t^2$$

$$E(X) = G'(1) = 0.7$$

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Recap

Topic 4 – key takeaways

1 Continuous

median solves $F(m) = 0.5$

2 t -inference

small samples use the t -distribution

3 χ^2

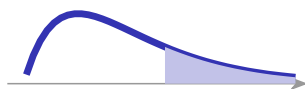
$\sum(O - E)^2/E$; fit & independence

4 PGF

$G(t) = E(t^X)$; $E(X) = G'(1)$

Check yourself

- 1 Which distribution for a small-sample mean test?
- 2 Write the chi-squared statistic.
- 3 What does the sign test compare?
- 4 How do you get $E(X)$ from a PGF?



Answers 1. the t -distribution 2. $\sum(O - E)^2/E$ 3. counts above/below a median
4. $E(X) = G'(1)$