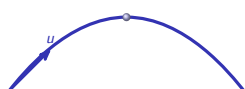


Further Mechanics

A-Level Further Mathematics • Topic 3

A-Level Further Mathematics



Further Mechanics

What we will cover

- 1 Projectiles
- 2 Rigid-body equilibrium
- 3 Circular motion
- 4 Hooke's law & elastic energy
- 5 Variable force & restitution
- 6 Recap

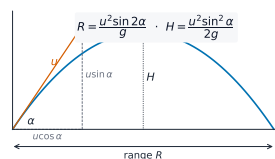


1 Projectiles and equilibrium

Further Mechanics

Projectiles and equilibrium

Projectile motion



Separate horizontal & vertical:

$$\text{range} = \frac{u^2 \sin 2\alpha}{g}, \quad H = \frac{u^2 \sin^2 \alpha}{2g}.$$

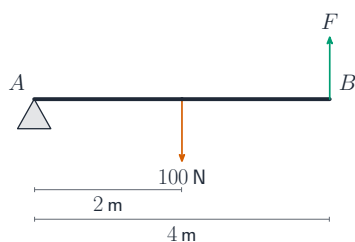
$$u = 20, \alpha = 30^\circ$$

$$\text{range} = 34.6 \text{ m}, H = 5 \text{ m}$$

Further Mechanics

Projectiles and equilibrium

Equilibrium of a rigid body



Moment 力矩 = force $\times$ perpendicular distance; weight acts at the **centre of mass** 质心.

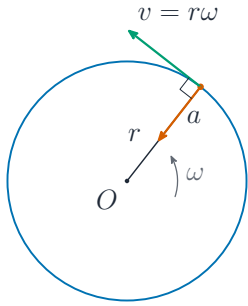
$$\text{Equilibrium: } \sum F = 0 \text{ and } \sum M = 0.$$

Beam 4 m, 100 N, pivot at A

$$F \times 4 = 100 \times 2 \Rightarrow F = 50 \text{ N}$$

2 Circular motion and elasticity

Circular motion



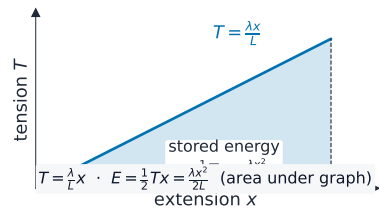
$v = r\omega$; the **centripetal acceleration** 向心加速度 points to the centre:

$$a = r\omega^2 = \frac{v^2}{r}$$

$$r = 2, \omega = 3 \text{ rad/s}$$

$$v = 6 \text{ m/s}, a = 18 \text{ m/s}^2$$

Hooke's law and elastic energy



$$T = \frac{\lambda x}{L}, \quad E = \frac{\lambda x^2}{2L}$$

λ is the **modulus of elasticity** 弹性模量.

$$L = 2, \lambda = 50, x = 0.5 \text{ m}$$

$$T = 12.5 \text{ N}, E = 3.125 \text{ J}$$

3

Variable force and momentum

Variable force and momentum

A position-dependent **variable force** 变力 uses

$$a = v \frac{dv}{dx},$$

turning Newton's law into a differential equation in v and x .

before



at rest

after



Restitution

Momentum is conserved; the bounciness is the **coefficient of restitution** 恢复系数:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}, \quad 0 \leq e \leq 1.$$

$e = 1$ elastic, $e = 0$ inelastic.

$$2 \text{ kg at } 5 \text{ hits still } 3 \text{ kg}, e = 0.5$$

$$v_A = 0.5, v_B = 3.0 \text{ m/s}$$

4

Recap

Topic 3 – key takeaways

1 Projectiles

range $u^2 \sin 2\alpha / g$; parabolic path

2 Circular

$v = r\omega$, $a = v^2/r$ toward the centre

3 Hooke

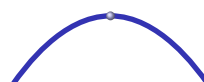
$T = \lambda x/L$; $E = \lambda x^2/2L$

4 Restitution

$e = \text{separation/approach speed}$

Check yourself

- 1 Direction of the centripetal acceleration?
- 2 Tension formula for an elastic string?
- 3 What is e for a perfectly elastic collision?
- 4 Which form of a suits a force depending on x ?



Answers 1. toward the centre 2. $T = \lambda x/L$ 3. $e = 1$ 4. $a = v dv/dx$