Further Mechanics

A-Level Further Mathematics ▪ Topic 3

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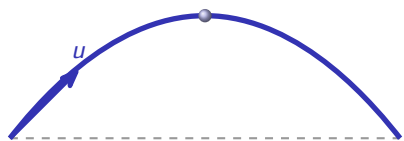
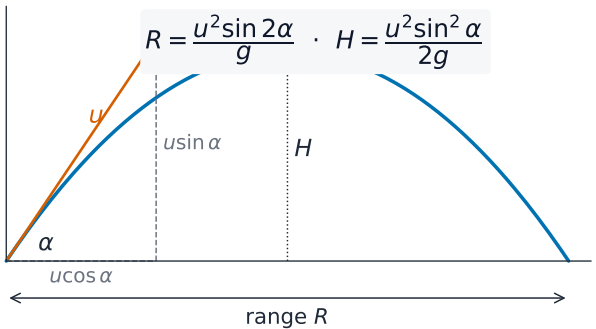


What we will cover

1. Projectiles
2. Rigid-body equilibrium
3. Circular motion
4. Hooke's law & elastic energy
5. Variable force & restitution
6. Recap

1 Projectiles and equilibrium

Projectile motion



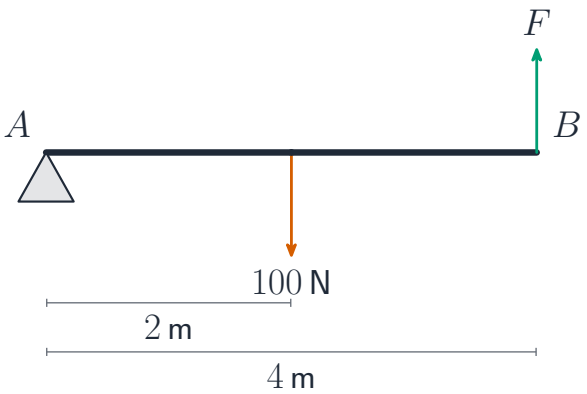
Separate horizontal & vertical:

$$\text{range} = \frac{u^2 \sin 2\alpha}{g}, \quad H = \frac{u^2 \sin^2 \alpha}{2g}.$$

$u = 20$, $\alpha = 30°$

range $= 34.6\,\text{m}$, $H = 5\,\text{m}$

Equilibrium of a rigid body



Moment 力矩 = force × perpendicular distance; weight acts at the **centre of mass** 质心.

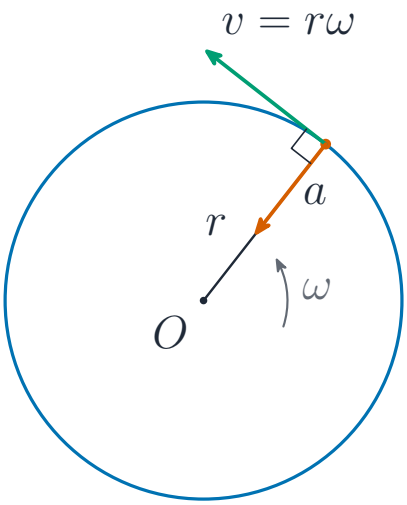
Equilibrium: $\sum F = 0$ and $\sum M = 0$.

Beam $4\,\mathrm{m}$, $100\,\mathrm{N}$, pivot at A

$F \times 4 = 100 \times 2 \Rightarrow F = 50\,\mathrm{N}$

2 Circular motion and elasticity

Circular motion



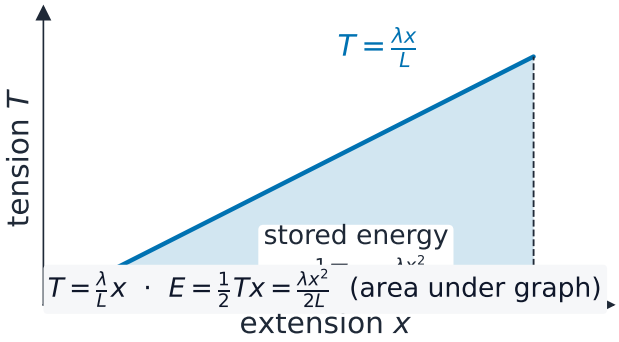
$v = r\omega$; the **centripetal acceleration** 向心加速度 points to the centre:

$$a = r\omega^2 = \frac{v^2}{r}.$$

$r = 2$, $\omega = 3\,\mathrm{rad/s}$

$v = 6\,\mathrm{m/s}$, $a = 18\,\mathrm{m/s^2}$

Hooke's law and elastic energy



$$T = \frac{\lambda x}{L}, \qquad E = \frac{\lambda x^2}{2L}.$$

λ is the **modulus of elasticity** 弹性模量.

$L = 2$, $\lambda = 50$, $x = 0.5\,\mathrm{m}$

$T = 12.5\,\mathrm{N}$, $E = 3.125\,\mathrm{J}$

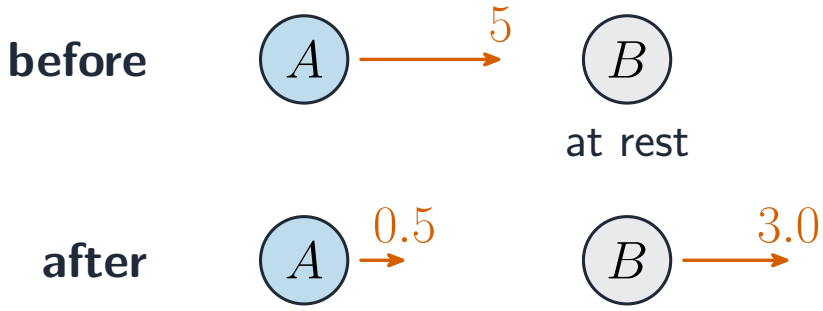
3 Variable force and momentum

Variable force and momentum

A position-dependent **variable force** 变力 uses

$$a = v\frac{dv}{dx},$$

turning Newton's law into a differential equation in v and x.



Restitution

Momentum is conserved; the bounciness is the **coefficient of restitution** 恢复系数:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}},\ 0 \leqslant e \leqslant 1.$$

$e = 1$ elastic, $e = 0$ inelastic.

$2\,\mathrm{kg}$ at 5 hits still $3\,\mathrm{kg}$, $e = 0.5$

$v_A = 0.5$, $v_B = 3.0\,\mathrm{m/s}$

4 Recap

Topic 3 – key takeaways

1 Projectiles

range $u^2 \sin 2\alpha / g$; parabolic path

2 Circular

$v = r\omega$, $a = v^2/r$ toward the centre

3 Hooke

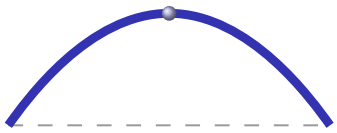
$T = \lambda x / L$; $E = \lambda x^2 / 2L$

4 Restitution

$e =$ separation/approach speed

Check yourself

1. Direction of the centripetal acceleration?
2. Tension formula for an elastic string?
3. What is e for a perfectly elastic collision?
4. Which form of a suits a force depending on x?



Answers **1.** toward the centre **2.** $T = \lambda x/L$ **3.** $e = 1$ **4.** $a = v\,dv/dx$