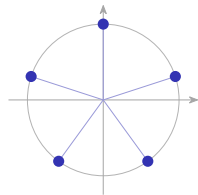


Further Pure Mathematics 2

A-Level Further Mathematics ■ Topic 2

A-Level Further Mathematics



Further Pure Mathematics 2

What we will cover

- 1 Hyperbolic functions
- 2 Eigenvalues & eigenvectors
- 3 Calculus & Maclaurin series
- 4 Complex numbers
- 5 Differential equations
- 6 Recap



1

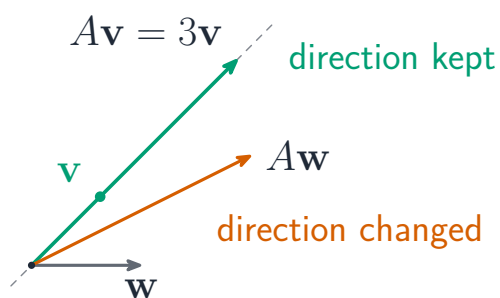
Hyperbolic and matrices

Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}, \quad \tanh x = \frac{\sinh x}{\cosh x}$$

Identities echo trig, with sign changes: $\cosh^2 x - \sinh^2 x = 1$. Derivatives:
 $(\sinh x)' = \cosh x$, $(\cosh x)' = \sinh x$.

Eigenvalues and eigenvectors



Characteristic equation 特征方程
 $\det(A - \lambda I) = 0$ gives the
eigenvalues 特征值; $Av = \lambda v$ the
eigenvectors 特征向量.

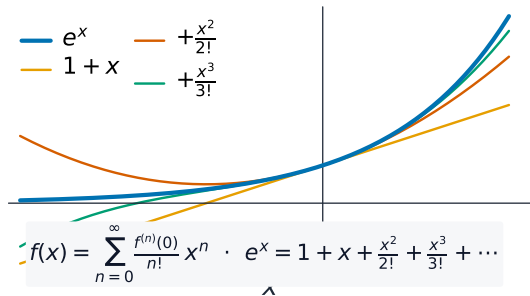
$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$(2 - \lambda)^2 - 1 = 0 \Rightarrow \lambda = 1, 3$$

Then $A = QDQ^{-1}$.

2 Calculus

Differentiation and Maclaurin series



Maclaurin series 麦克劳林级数:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

e.g. $e^x = 1 + x + \frac{x^2}{2!} + \dots$ – more terms, wider match.

Integration

Standard integrals:

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C,$$

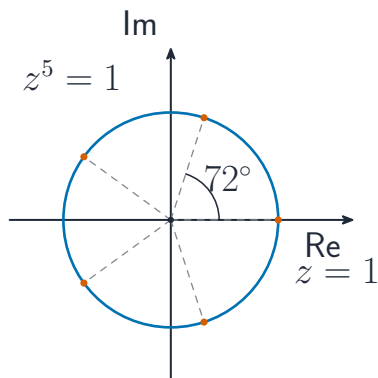
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C.$$

$$\int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1} \frac{x}{2} + C$$

A **reduction formula** 递推公式 links I_n to I_{n-1} .

3 Complex and DEs

Complex numbers – de Moivre

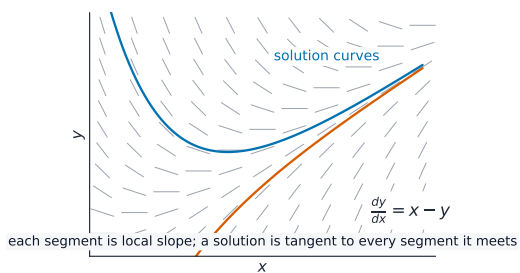


De Moivre 棣莫弗定理:

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta.$$

Gives multiple-angle formulae and the n **roots of unity** 单位根 (equally spaced).

Differential equations



First-order linear: multiply by the **integrating factor** 积分因子
 $\mu = e^{\int P dx}$.

$$\frac{dy}{dx} + 2y = e^x$$

$$\mu = e^{2x} \Rightarrow y = \frac{1}{3}e^x + Ce^{-2x}$$

Constant-coefficient:
complementary function 余函数
 + **particular integral** 特积分.

4

Recap

Topic 2 – key takeaways

1 Hyperbolic

$$\cosh^2 - \sinh^2 = 1$$

2 Eigenvalues

$$\det(A - \lambda I) = 0; A = QDQ^{-1}$$

3 Complex

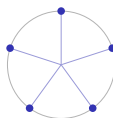
de Moivre; roots of unity

4 DEs

integrating factor; CF + PI

Check yourself

- 1 Simplify $\cosh^2 x - \sinh^2 x$.
- 2 What equation gives the eigenvalues?
- 3 State de Moivre's theorem.
- 4 What multiplies a first-order linear DE?



Answers 1. 1 2. $\det(A - \lambda I) = 0$ 3. $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ 4. the integrating factor