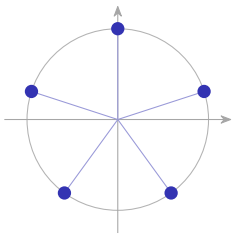
Further Pure Mathematics 2

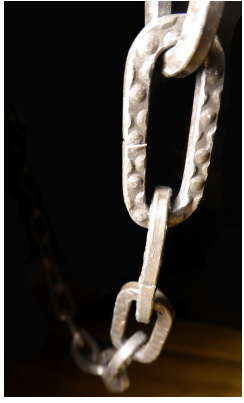
A-Level Further Mathematics ▪ Topic 2

A-Level Further Mathematics



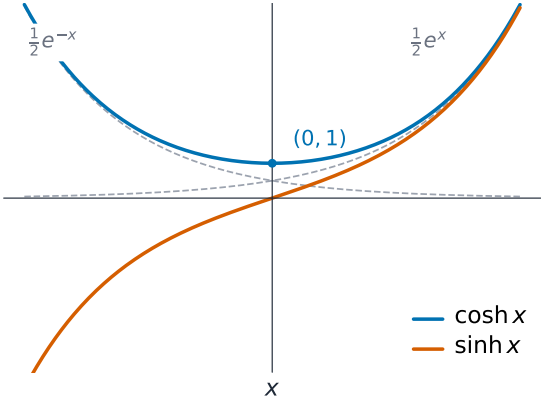
What we will cover

1. Hyperbolic functions
2. Eigenvalues & eigenvectors
3. Calculus & Maclaurin series
4. Complex numbers
5. Differential equations
6. Recap



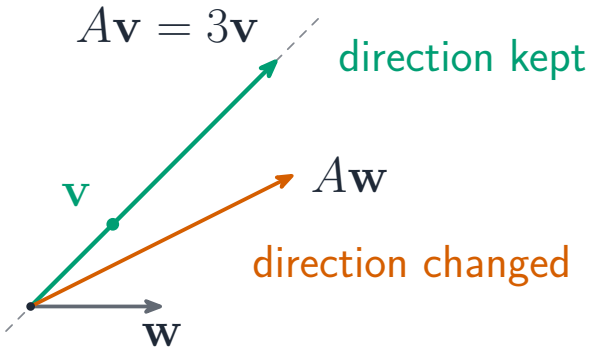
1 Hyperbolic and matrices

Hyperbolic functions



Fractal

Eigenvalues and eigenvectors



Characteristic equation 特征方程 $\det(A - \lambda I) = 0$ gives the **eigenvalues** 特征值; $A\mathbf{v} = \lambda\mathbf{v}$ the **eigenvectors** 特征向量.

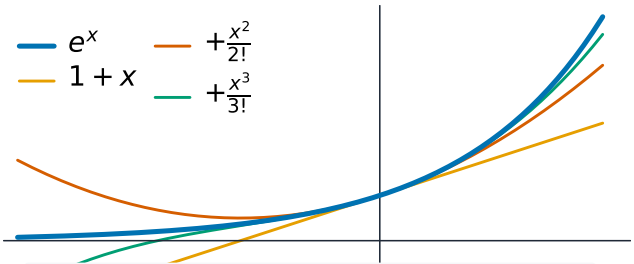
$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$(2 - \lambda)^2 - 1 = 0 \Rightarrow \lambda = 1, 3$

Then $A = QDQ^{-1}$.

2 Calculus

Differentiation and Maclaurin series



$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \cdot \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Maclaurin series 麦克劳林级数:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

e.g. $e^x = 1 + x + \frac{x^2}{2!} + \cdots$ – more terms, wider match.

Integration

Standard integrals:

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C,$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C.$$

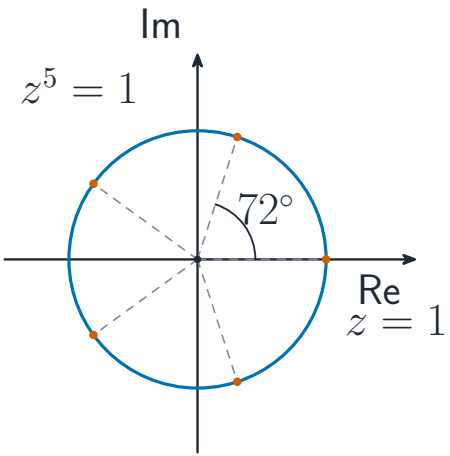
$\int \frac{dx}{\sqrt{4-x^2}}$

$= \sin^{-1}\frac{x}{2} + C$

A **reduction formula** 递推公式 links I_n to I_{n-1}.

3 Complex and DEs

Complex numbers – de Moivre

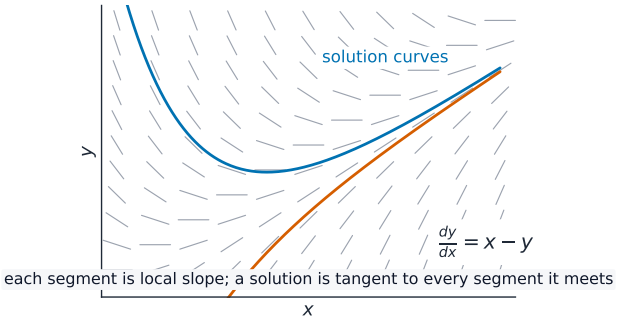


De Moivre 棣莫弗定理:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

Gives multiple-angle formulae and the n **roots of unity** 单位根 (equally spaced).

Differential equations



each segment is local slope; a solution is tangent to every segment it meets

First-order linear: multiply by the **integrating factor** 积分因子 $\mu = e^{\int P\,dx}$.

$$\frac{dy}{dx} + 2y = e^x$$

$$\mu = e^{2x} \Rightarrow y = \tfrac{1}{3}e^x + Ce^{-2x}$$

Constant-coefficient: **complementary function** 余函数 + **particular integral** 特积分.

4 Recap

Topic 2 – key takeaways

1 Hyperbolic

$\cosh^2 - \sinh^2 = 1$

2 Eigenvalues

$\det(A - \lambda I) = 0$; $A = QDQ^{-1}$

3 Complex

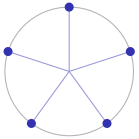
de Moivre; roots of unity

4 DEs

integrating factor; CF + PI

Check yourself

1. Simplify $\cosh^2 x - \sinh^2 x$.
2. What equation gives the eigenvalues?
3. State de Moivre's theorem.
4. What multiplies a first-order linear DE?



Answers **1.** 1 **2.** $\det(A - \lambda I) = 0$ **3.** $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ **4.** the integrating factor