# Further Pure Mathematics 1

A-Level Further Mathematics ▪ Topic 1

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## What we will cover



# 1 Roots and series

## Roots and summation of series

Roots $\leftrightarrow$ coefficients (quadratic):

$$\alpha+\beta=-\frac{b}{a},\quad \alpha\beta=\frac{c}{a}.$$

Standard sums: $\sum r=\frac{1}{2}n(n+1)$, $\sum r^2=\frac{1}{6}n(n+1)(2n+1)$.

**$\sum_{r=1}^{n}(2r+1)$**

$$=2\cdot\tfrac{1}{2}n(n+1)+n=n(n+2)$$

**Method of differences** 差分法 cancels middle terms.

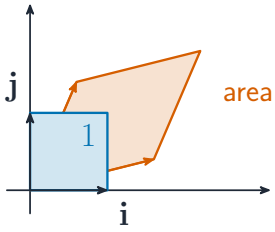


## Rational functions

If the numerator's degree is one more than the denominator, divide first – the quotient is an oblique asymptote. Vertical asymptotes are where the denominator is zero (and the numerator is not). Horizontal: compare degrees.

# 2 Matrices

## Matrices




area $=$ det $= 2$

$\det A = ad - bc$ (the area scale factor).

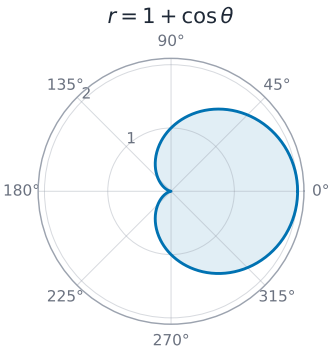
$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

A $2 \times 2$ matrix is a **transformation** 几何变换; $AB$ means "do $B$ then $A$".

# 3 Polar and vectors



## Polar coordinates



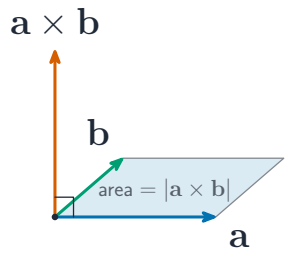

*the cardioid* $r = 1 + \cos\theta$

**Polar** 极坐标 $(r, \theta)$: $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$.

$$\text{area} = \tfrac{1}{2}\int r^2\, d\theta.$$

e.g. $r = 4\cos\theta \Rightarrow (x-2)^2 + y^2 = 4$.

## Vectors




A **plane** 平面: $\mathbf{r} \cdot \mathbf{n} = p$. The **vector product** 向量积

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\, \hat{\mathbf{n}}$$

is perpendicular to both; its length is the parallelogram's area.



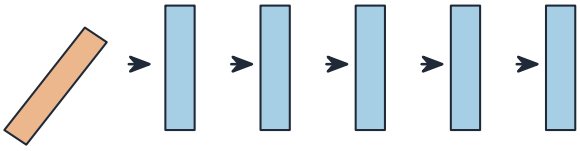
## Proof by induction

**Induction** 数学归纳法:

1. base case $n = 1$
2. inductive step: true for $k \Rightarrow$ true for $k + 1$

Then it holds for every $n$.

so every domino falls — true for all $n$



# 4 Recap

## Topic 1 – key takeaways

**1 Roots**

sums & products link to coefficients

**2 Matrices**

$\det = ad - bc$; transformations

**3 Polar**

$x = r\cos\theta$; area $\frac{1}{2}\int r^2 d\theta$

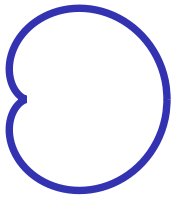
**4 Induction**

base case + inductive step



## Check yourself

1. For $ax^2 + bx + c = 0$, what is $\alpha\beta$?
2. What is $\det\left(\begin{smallmatrix}3 & 1\\ 2 & 4\end{smallmatrix}\right)$?
3. Convert $(r, \theta)$ to the $x$-coordinate.
4. The two steps of proof by induction?



**Answers** **1.** $c/a$ **2.** $10$ **3.** $x = r\cos\theta$ **4.** base case, then inductive step