

Further Pure Mathematics 1

A-Level Further Mathematics • Topic 1

A-Level Further Mathematics



Further Pure Mathematics 1

What we will cover

- 1 Roots & series
- 2 Rational functions
- 3 Matrices
- 4 Polar coordinates
- 5 Vectors & induction
- 6 Recap



1

Roots and series

Further Pure Mathematics 1

└─ Roots and series

Roots and summation of series

Roots $\leftrightarrow$ coefficients (quadratic):

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

Standard sums: $\sum r = \frac{1}{2}n(n+1)$,
 $\sum r^2 = \frac{1}{6}n(n+1)(2n+1)$.

$$\begin{aligned} \sum_{r=1}^n (2r+1) \\ = 2 \cdot \frac{1}{2}n(n+1) + n = n(n+2) \end{aligned}$$

Method of differences 差分法 cancels middle terms.

Further Pure Mathematics 1

└─ Roots and series

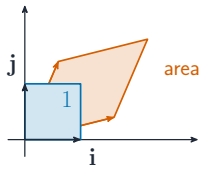
Rational functions

If the numerator's degree is one more than the denominator, divide first – the quotient is an **oblique** asymptote. Vertical asymptotes are where the denominator is zero (and the numerator is not). Horizontal: compare degrees.

2

Matrices

Matrices



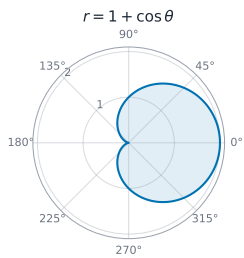
$\det A = ad - bc$ (the area scale factor).

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

A 2×2 matrix is a **transformation** 几何变换; AB means "do B then A ".

3 Polar and vectors

Polar coordinates



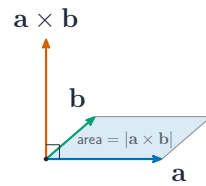
the cardioid $r = 1 + \cos \theta$

Polar 极坐标 (r, θ) : $x = r \cos \theta$, $y = r \sin \theta$,
 $r^2 = x^2 + y^2$.

$$\text{area} = \frac{1}{2} \int r^2 d\theta.$$

e.g. $r = 4 \cos \theta \Rightarrow (x - 2)^2 + y^2 = 4$.

Vectors



A **plane** 平面: $\mathbf{r} \cdot \mathbf{n} = p$. The **vector product** 向量积

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}}$$

is perpendicular to both; its length is the parallelogram's area.

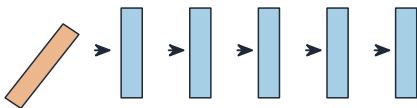
Proof by induction

Induction 数学归纳法:

- 1 base case $n = 1$
- 2 inductive step: true for $k \Rightarrow$ true for $k + 1$

Then it holds for every n .

so every domino falls — true for all n



4 Recap

Topic 1 – key takeaways

1 Roots

sums & products link to coefficients

2 Matrices

$\det = ad - bc$; transformations

3 Polar

$x = r \cos \theta$; area $\frac{1}{2} \int r^2 d\theta$

4 Induction

base case + inductive step

Check yourself

1 For $ax^2 + bx + c = 0$, what is $\alpha\beta$?

2 What is $\det \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$?

3 Convert (r, θ) to the x -coordinate.

4 The two steps of proof by induction?



Answers 1. c/a 2. 10 3. $x = r \cos \theta$ 4. base case, then inductive step