A-Level Further Mathematics

Revision Sheet · 考前速记

Learn these by heart before the exam. 考试前把这些内容背熟。

Complex numbers

$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$ 复数

$|z| = \sqrt{x^2+y^2}, \quad \arg z = \arctan\frac{y}{x}$ 模与辐角

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ de Moivre's theorem 棣莫弗定理

$z = re^{i\theta}$: $z^{1/n} = r^{1/n}e^{i(\theta+2k\pi)/n}$ n-th roots are equally spaced around a circle

Matrices

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$ 行列式

$M^{-1} = \frac{1}{\det M}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ singular (no inverse) when $\det M = 0$ 逆矩阵

Eigenvalues and eigenvectors 特征值与特征向量: $M\mathbf{v} = \lambda\mathbf{v}$; solve $\det(M - \lambda I) = 0$ for λ, then find each $\mathbf{v}$

$M = PDP^{-1}, \quad M^n = PD^nP^{-1}$ diagonalisation; D holds the eigenvalues

$\det M$ *is the area/volume scale factor of the transformation.*

Series and induction

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

Method of differences 差分法: write $u_r = f(r) - f(r+1)$ so the sum telescopes and most terms cancel

Proof by induction 数学归纳法: prove for $n = 1$; assume true for $n = k$; prove it for $n = k+1$; conclude true for all n

Calculus and polar

Maclaurin: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$ 麦克劳林级数

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + c$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + c$

Area (polar) $= \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$ 极坐标面积

$\cosh x = \frac{e^x+e^{-x}}{2}, \quad \sinh x = \frac{e^x-e^{-x}}{2}$ 双曲函数

$\cosh^2 x - \sinh^2 x = 1$

Differential equations

$\frac{dy}{dx} + P(x)y = Q(x)$: IF $= e^{\int P\,dx}$ first-order linear; multiply through by the integrating factor 积分因子

Second-order (constant coefficients) 二阶常系数: solve the auxiliary equation $am^2 + bm + c = 0$; the general solution = complementary function + particular integral

Real distinct roots $\Rightarrow Ae^{m_1x} + Be^{m_2x}$*; complex roots* $\Rightarrow e^{px}(A\cos qx + B\sin qx)$.

Further mechanics and statistics

Momentum $p = mv$, impulse $= \Delta p$ 动量与冲量

$e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$ Newton's law of restitution; $0 \le e \le 1$ 恢复系数

$E_{\text{elastic}} = \dfrac{\lambda x^2}{2l}$ energy stored in an elastic string/spring

$\chi^2 = \sum \dfrac{(O-E)^2}{E}$ goodness of fit; compare with the critical value 卡方检验

Type I and Type II error 第一类与第二类错误: Type I: reject a true H_0; Type II: accept a false H_0