1. Further Pure Mathematics 1

Weeks 1–13

进阶纯数学 1

They should be able to

1. • recall and use the relations between the **roots** and **coefficients** of polynomial equations
2. • use a **substitution** to obtain an equation whose roots are related in a simple way to those of the original equation
3. • sketch graphs of simple **rational functions**, including the determination of **oblique asymptotes**, in cases where the degree of the numerator and the denominator are at most 2
4. • understand and use relationships between the graphs of $y = f(x)$, $y^2 = f(x)$, $y = \frac{1}{f(x)}$, $y = \
5. • use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums
6. • use the **method of differences** to obtain the sum of a finite series
7. • recognise, by direct consideration of a sum to n terms, when a series is **convergent**, and find the **sum to infinity** in such cases
8. carry out operations of **matrix addition**, subtraction and multiplication, and recognise the terms **zero matrix** and **identity (or unit) matrix**
9. recall the meaning of the terms **singular** and **non-singular** as applied to square matrices and, for 2×2 and 3×3 matrices, evaluate **determinants** and find **inverses** of non-singular matrices
10. understand and use the result, for non-singular matrices, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
11. understand the use of 2×2 matrices to represent certain geometric **transformations** in the x-y plane, in particular: – understand the relationship between the transformations represented by $\mathbf{A}$ and $\mathbf{A}^{-1}$ – recognise that the matrix product $\mathbf{AB}$ represents the transformation that results from the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$ – recall how the **area scale factor** of a transformation is related to the determinant of the corresponding matrix – find the matrix that represents a given transformation or sequence of transformations
12. understand the meaning of **invariant** as applied to points and lines in the context of transformations represented by matrices, and solve simple problems involving **invariant points** and **invariant lines**.
13. understand the relations between **Cartesian** and **polar coordinates**, and convert equations of curves from Cartesian to polar form and vice versa
14. sketch simple **polar curves**, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals
15. recall the formula $\frac{1}{2} \int r^2 \mathrm{d}\theta$ for the area of a sector, and use this formula in simple cases.
16. use the equation of a **plane** in any of the forms $ax + by + cz = d$ or $\mathbf{r.n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and convert equations of planes from one form to another as necessary in solving problems
17. recall that the **vector product** $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\
18. use equations of lines and planes, together with **scalar and vector products** where appropriate, to solve problems concerning distances, angles and intersections, including: – determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists – finding the foot of the perpendicular from a point to a plane – finding the angle between a line and a plane, and the angle between two planes – finding an equation for the line of intersection of two planes – calculating the shortest distance between two **skew lines** – finding an equation for the common perpendicular to two skew lines.
19. use the method of **mathematical induction** to establish a given result
20. recognise situations where **conjecture** based on a limited trial followed by **inductive proof** is a useful strategy, and carry this out in simple cases.

Stage	Use	Your notes
Starter 10 min · retrieval practice	Topic quiz 01 (10 questions, answers on sheet 2)	
Teach the new content	Handout 01 Slide deck 01 Vocabulary list 01	
Practice guided then independent	Exercise sheets 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7 Past-paper sheets 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7	
Check before they leave	Unit test 1 (with mark scheme)	
Homework set from the Library	Any unused exercise sheet above Holiday homework pack	

2. Further Pure Mathematics 2 — Weeks 14–30

进阶纯数学 2

They should be able to

1. understand the definitions of the **hyperbolic functions** $\sinh x$, $\cosh x$, $\tanh x$, $\text{sech } x$, $\text{cosech } x$, $\text{coth } x$ in terms of the exponential function
2. sketch the graphs of **hyperbolic functions**
3. prove and use identities involving **hyperbolic functions**
4. understand and use the definitions of the **inverse hyperbolic functions** and derive and use the logarithmic forms
5. formulate a problem involving the solution of 3 linear simultaneous equations in 3 unknowns as a problem involving the solution of a **matrix equation**, or vice versa
6. understand the cases that may arise concerning the consistency or inconsistency of 3 linear simultaneous equations, relate them to the **singularity** or otherwise of the corresponding matrix, solve consistent systems, and interpret geometrically in terms of lines and planes
7. understand the terms '**characteristic equation**', '**eigenvalue**' and '**eigenvector**', as applied to square matrices
8. find **eigenvalues** and **eigenvectors** of 2×2 and 3×3 matrices
9. express a square matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a **diagonal matrix** of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use this expression
10. use the fact that a square matrix satisfies its own **characteristic equation**.
11. • differentiate **hyperbolic functions** and differentiate $\sin^{-1}x$, $\cos^{-1}x$, $\sinh^{-1}x$, $\cosh^{-1}x$ and $\tanh^{-1}x$
12. • obtain an expression for $\frac{\text{d}^2y}{\text{d}x^2}$ in cases where the relation between x and y is defined **implicitly** or **parametrically**
13. • derive and use the first few terms of a **Maclaurin's series** for a function.
14. • integrate **hyperbolic functions** and recognise integrals of functions of the form $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{\sqrt{x^2 + a^2}}$ and $\frac{1}{\sqrt{x^2 - a^2}}$, and integrate associated functions using **trigonometric substitutions** or **hyperbolic substitutions** as appropriate
15. • derive and use **reduction formulae** for the evaluation of definite integrals
16. • understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities or limits concerning sums
17. • use **integration** to find – arc lengths for curves with equations in Cartesian coordinates, including the use of a parameter, or in polar coordinates – surface areas of revolution about one of the axes for curves with equations in Cartesian coordinates, including the use of a parameter.
18. • understand **de Moivre's theorem**, for a positive or negative integer exponent, in terms of the geometrical effect of multiplication and division of **complex numbers**
19. • prove **de Moivre's theorem** for a positive integer exponent
20. • use **de Moivre's theorem** for a positive or negative rational exponent
21. – to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle
22. – to express powers of $\sin \theta$ and $\cos \theta$ in terms of multiple angles
23. – in the summation of series
24. – in finding and using the nth **roots of unity**.
25. • find an **integrating factor** for a first order linear **differential equation**, and use an **integrating factor** to find the general solution
26. • recall the meaning of the terms '**complementary function**' and '**particular integral**' in the context of linear **differential equations**, and recall that the general solution is the sum of the **complementary function** and a **particular integral**
27. find the **complementary function** for a first or second order linear differential equation with constant coefficients

28. recall the form of, and find, a **particular integral** for a first or second order linear differential equation in the cases where a polynomial or ae^{bx} or $a\cos px + b\sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral
29. use a given **substitution** to reduce a differential equation to a first or second order linear equation with constant coefficients or to a first order equation with **separable variables**
30. use **initial conditions** to find a **particular solution** to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.

Stage	Use	Your notes
Starter 10 min · retrieval practice	Topic quiz 02 (10 questions, answers on sheet 2)	
Teach the new content	Handout 02 Slide deck 02 Vocabulary list 02	
Practice guided then independent	Exercise sheets 2.1, 2.2, 2.3, 2.4, 2.5, 2.6 Past-paper sheets 2.1, 2.2, 2.3, 2.4, 2.5, 2.6	
Check before they leave	Unit test 2 (with mark scheme)	
Homework set from the Library	Any unused exercise sheet above Holiday homework pack	

3. Further Mechanics Weeks 31–43

进阶力学

They should be able to

1. • model the motion of a **projectile** as a particle moving with constant acceleration and understand any limitations of the model
2. • use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached
3. • derive and use the **Cartesian equation** of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.
4. • calculate the **moment** of a force about a point
5. • use the result that the effect of gravity on a **rigid body** is equivalent to a single force acting at the **centre of mass** of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry
6. • use given information about the position of the centre of mass of a triangular **lamina** and other simple shapes
7. • determine the position of the centre of mass of a **composite body** by considering an equivalent system of particles
8. • use the principle that if a rigid body is in **equilibrium** under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this
9. • solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving **toppling** or **sliding**.
10. understand the concept of **angular speed** for a particle moving in a circle, and use the relation $v = r\omega$
11. understand that the **acceleration** of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$.
12. solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed
13. solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy.
14. use **Hooke's law** as a model relating the force in an elastic string or spring to the extension or compression, and understand the term **modulus of elasticity**
15. use the formula for the **elastic potential energy** stored in a string or spring
16. solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.
17. solve problems which can be modelled as the linear motion of a particle under the action of a **variable force**, by setting up and solving an appropriate **differential equation**.
18. recall **Newton's experimental law** and the definition of the **coefficient of restitution**, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms '**perfectly elastic**' ($e = 1$) and '**inelastic**' ($e = 0$)
19. use **conservation of linear momentum** and/or **Newton's experimental law** to solve problems that may be modelled as the **direct** or **oblique impact** of two smooth spheres, or the **direct** or **oblique impact** of a smooth sphere with a fixed surface.

Stage	Use	Your notes
Starter 10 min · retrieval practice	Topic quiz 03 (10 questions, answers on sheet 2) Starter quiz 3 — past-paper questions	
Teach the new content	Handout 03 Slide deck 03 Vocabulary list 03	

Stage	Use	Your notes
Practice guided then independent	Exercise sheets 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 Past-paper sheets 3.1, 3.2, 3.3, 3.4, 3.5, 3.6	
Check before they leave	Unit test 3 (with mark scheme)	
Homework set from the Library	Any unused exercise sheet above Holiday homework pack	

4. Further Probability & Statistics

Weeks 44–56

进阶概率与统计

They should be able to

1. use a **probability density function** which may be defined piecewise
2. use the general result $\text{E}(g(X)) = \int f(x)g(x) \, \mathrm{d}x$ where $f(x)$ is the **probability density function** of the continuous random variable X and $g(X)$ is a function of X
3. understand and use the relationship between the **probability density function** (PDF) and the **cumulative distribution function** (CDF), and use either to evaluate probabilities or percentiles
4. use **cumulative distribution functions** (CDFs) of related variables in simple cases.
5. formulate hypotheses and apply a **hypothesis test** concerning the population mean using a small sample drawn from a normal population of unknown variance, using a **t-test**
6. calculate a **pooled estimate** of a population variance from two samples
7. formulate hypotheses concerning the difference of population means, and apply, as appropriate: - a **2-sample t-test** - a **paired sample t-test** - a test using a **normal distribution**
8. determine a **confidence interval** for a population mean, based on a small sample from a normal population with unknown variance, using a **t-distribution**
9. determine a **confidence interval** for a difference of population means, using a **t-distribution** or a **normal distribution**, as appropriate.
10. fit a theoretical distribution, as prescribed by a given hypothesis, to given data
11. use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding **goodness of fit** analysis
12. use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a **contingency table**.
13. understand the idea of a **non-parametric test** and appreciate situations in which such a test might be useful
14. understand the basis of the **sign test**, the **Wilcoxon signed-rank test** and the **Wilcoxon rank-sum test**
15. use a single-sample **sign test** and a single-sample **Wilcoxon signed-rank test** to test a hypothesis concerning a population **median**
16. use a paired-sample **sign test**, a **Wilcoxon matched-pairs signed-rank test** and a **Wilcoxon rank-sum test**, as appropriate, to test for identity of populations.
17. understand the concept of a **probability generating function** (PGF) and construct and use the PGF for given distributions
18. use formulae for the mean and variance of a discrete random variable in terms of its PGF, and use these formulae to calculate the mean and variance of a given probability distribution
19. use the result that the PGF of the sum of independent random variables is the product of the PGFs of those random variables.

Stage	Use	Your notes
Starter 10 min · retrieval practice	Topic quiz 04 (10 questions, answers on sheet 2)	
Teach the new content	Handout 04 Slide deck 04 Vocabulary list 04	
Practice guided then independent	Exercise sheets 4.1, 4.2, 4.3, 4.4, 4.5 Past-paper sheets 4.1, 4.2, 4.3, 4.4, 4.5	

Stage	Use	Your notes
Check before they leave	Unit test 4 (with mark scheme)	
Homework set from the Library	Any unused exercise sheet above Holiday homework pack	