A-Level Further Mathematics

Syllabus Checklist · 大纲自查表

Tick one box per statement: ☐ not yet ☐ nearly ☐ confident. 每条打一个勾：尚未掌握 / 快会了 / 有把握。

1. Further Pure Mathematics 1 进阶纯数学 1

1.1 Roots of polynomial equations 多项式方程的根

☐☐☐ • recall and use the relations between the **roots** and **coefficients** of polynomial equations

☐☐☐ • use a **substitution** to obtain an equation whose roots are related in a simple way to those of the original equation

1.2 Rational functions and graphs 有理函数与图像

☐☐☐ • sketch graphs of simple **rational functions**, including the determination of **oblique asymptotes**, in cases where the degree of the numerator and the denominator are at most 2

☐☐☐ • understand and use relationships between the graphs of $y = f(x)$, $y^2 = f(x)$, $y = \frac{1}{f(x)}$, y = \

1.3 Summation of series 级数求和

☐☐☐ • use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums

☐☐☐ • use the **method of differences** to obtain the sum of a finite series

☐☐☐ • recognise, by direct consideration of a sum to n terms, when a series is **convergent**, and find the **sum to infinity** in such cases

1.4 Matrices 矩阵

☐☐☐ carry out operations of **matrix addition**, subtraction and multiplication, and recognise the terms **zero matrix** and **identity (or unit) matrix**

☐☐☐ recall the meaning of the terms **singular** and **non-singular** as applied to square matrices and, for 2×2 and 3×3 matrices, evaluate **determinants** and find **inverses** of non-singular matrices

☐☐☐ understand and use the result, for non-singular matrices, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

☐☐☐ understand the use of 2×2 matrices to represent certain geometric **transformations** in the x-y plane, in particular: – understand the relationship between the transformations represented by **A** and $\mathbf{A}^{-1}$ – recognise that the matrix product **AB** represents the transformation that results from the transformation represented by **B** followed by the transformation represented by **A** – recall how the **area scale factor** of a transformation is related to the determinant of the corresponding matrix – find the matrix that represents a given transformation or sequence of transformations

☐☐☐ understand the meaning of **invariant** as applied to points and lines in the context of transformations represented by matrices, and solve simple problems involving **invariant points** and **invariant lines**.

1.5 Polar coordinates 极坐标

☐☐☐ understand the relations between **Cartesian** and **polar coordinates**, and convert equations of curves from Cartesian to polar form and vice versa

☐☐☐ sketch simple **polar curves**, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals

☐☐☐ recall the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$ for the area of a sector, and use this formula in simple cases.

1.6 Vectors 向量

☐☐☐ use the equation of a **plane** in any of the forms $ax + by + cz = d$ or $\mathbf{r}.\mathbf{n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and convert equations of planes from one form to another as necessary in solving problems

☐☐☐ recall that the **vector product a** $\times$ **b** of two vectors can be expressed either as \

☐☐☐ use equations of lines and planes, together with **scalar and vector products** where appropriate, to solve problems concerning distances, angles and intersections, including: – determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists – finding the foot of the perpendicular from a point to a plane – finding the angle between a line and a plane, and the angle between two planes – finding an equation for the line of intersection of two planes – calculating the shortest distance between two **skew lines** – finding an equation for the common perpendicular to two skew lines.

1.7 Proof by induction 数学归纳法证明

☐☐☐ use the method of **mathematical induction** to establish a given result

☐☐☐ recognise situations where **conjecture** based on a limited trial followed by **inductive proof** is a useful strategy, and carry this out in simple cases.

2. Further Pure Mathematics 2 进阶纯数学 2

2.1 Hyperbolic functions 双曲函数

☐☐☐ understand the definitions of the **hyperbolic functions** $\sinh x$, $\cosh x$, $\tanh x$, sech x, cosech x, coth x in terms of the exponential function

☐☐☐ sketch the graphs of **hyperbolic functions**

☐☐☐ prove and use identities involving **hyperbolic functions**

☐☐☐ understand and use the definitions of the **inverse hyperbolic functions** and derive and use the logarithmic forms

2.2 Matrices 矩阵

☐☐☐ formulate a problem involving the solution of 3 linear simultaneous equations in 3 unknowns as a problem involving the solution of a **matrix equation**, or vice versa

☐☐☐ understand the cases that may arise concerning the consistency or inconsistency of 3 linear simultaneous equations, relate them to the **singularity** or otherwise of the corresponding matrix, solve consistent systems, and interpret geometrically in terms of lines and planes

☐☐☐ understand the terms '**characteristic equation**', '**eigenvalue**' and '**eigenvector**', as applied to square matrices

☐☐☐ find **eigenvalues** and **eigenvectors** of 2×2 and 3×3 matrices

☐☐☐ express a square matrix in the form $\mathbf{QDQ}^{-1}$, where **D** is a **diagonal matrix** of eigenvalues and **Q** is a matrix whose columns are eigenvectors, and use this expression

☐☐☐ use the fact that a square matrix satisfies its own **characteristic equation**.

2.3 Differentiation 微分

☐☐☐ • differentiate **hyperbolic functions** and differentiate $\sin^{-1} x$, $\cos^{-1} x$, $\sinh^{-1} x$, $\cosh^{-1} x$ and $\tanh^{-1} x$

☐☐☐ • obtain an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in cases where the relation between x and y is defined **implicitly** or **parametrically**

☐☐☐ • derive and use the first few terms of a **Maclaurin's series** for a function.

2.4 Integration 积分

☐☐☐ • integrate **hyperbolic functions** and recognise integrals of functions of the form $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{\sqrt{x^2+a^2}}$ and $\frac{1}{\sqrt{x^2-a^2}}$, and integrate associated functions using **trigonometric substitutions** or **hyperbolic substitutions** as appropriate

☐☐☐ • derive and use **reduction formulae** for the evaluation of definite integrals

☐☐☐ • understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities or limits concerning sums

☐☐☐ • use **integration** to find – arc lengths for curves with equations in Cartesian coordinates, including the use of a parameter, or in polar coordinates – surface areas of revolution about one of the axes for curves with equations in Cartesian coordinates, including the use of a parameter.

2.5 Complex numbers 复数

☐☐☐ • understand **de Moivre's theorem**, for a positive or negative integer exponent, in terms of the geometrical effect of multiplication and division of **complex numbers**

☐☐☐ • prove **de Moivre's theorem** for a positive integer exponent

☐☐☐ • use **de Moivre's theorem** for a positive or negative rational exponent

☐☐☐ – to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle

☐☐☐ – to express powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles

☐☐☐ – in the summation of series

☐☐☐ – in finding and using the nth **roots of unity**.

2.6 Differential equations 微分方程

☐☐☐ • find an **integrating factor** for a first order linear **differential equation**, and use an **integrating factor** to find the general solution

☐☐☐ • recall the meaning of the terms '**complementary function**' and '**particular integral**' in the context of linear **differential equations**, and recall that the general solution is the sum of the **complementary function** and a **particular integral**

☐☐☐ find the **complementary function** for a first or second order linear differential equation with constant coefficients

☐☐☐ recall the form of, and find, a **particular integral** for a first or second order linear differential equation in the cases where a polynomial or ae^{bx} or $a\cos px + b\sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral

☐☐☐ use a given **substitution** to reduce a differential equation to a first or second order linear equation with constant coefficients or to a first order equation with **separable variables**

☐☐☐ use **initial conditions** to find a **particular solution** to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.

3. Further Mechanics 进阶力学

3.1 Motion of a projectile 抛体运动

☐☐☐ • model the motion of a **projectile** as a particle moving with constant acceleration and understand any limitations of the model

☐☐☐ • use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached

☐☐☐ • derive and use the **Cartesian equation** of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.

3.2 Equilibrium of a rigid body 刚体的平衡

☐☐☐ • calculate the **moment** of a force about a point

☐☐☐ • use the result that the effect of gravity on a **rigid body** is equivalent to a single force acting at the **centre of mass** of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry

☐☐☐ • use given information about the position of the centre of mass of a triangular **lamina** and other simple shapes

☐☐☐ • determine the position of the centre of mass of a **composite body** by considering an equivalent system of particles

☐☐☐ • use the principle that if a rigid body is in **equilibrium** under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this

☐☐☐ • solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving **toppling** or **sliding**.

3.3 Circular motion 圆周运动

☐☐☐ understand the concept of **angular speed** for a particle moving in a circle, and use the relation $v = r\omega$

☐☐☐ understand that the **acceleration** of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$.

☐☐☐ solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed

☐☐☐ solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy.

3.4 Hooke's law 胡克定律

☐☐☐ use **Hooke's law** as a model relating the force in an elastic string or spring to the extension or compression, and understand the term **modulus of elasticity**

☐☐☐ use the formula for the **elastic potential energy** stored in a string or spring

☐☐☐ solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.

3.5 Linear motion under a variable force 变力作用下的直线运动

☐☐☐ solve problems which can be modelled as the linear motion of a particle under the action of a **variable force**, by setting up and solving an appropriate **differential equation**.

3.6 Momentum 动量

☐☐☐ recall **Newton's experimental law** and the definition of the **coefficient of restitution**, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms '**perfectly elastic**' ($e = 1$) and '**inelastic**' ($e = 0$)

☐☐☐ use **conservation of linear momentum** and/or **Newton's experimental law** to solve problems that may be modelled as the **direct** or **oblique impact** of two smooth spheres, or the **direct** or **oblique impact** of a smooth sphere with a fixed surface.

4.1 Continuous random variables 连续随机变量

- ☐☐☐ use a **probability density function** which may be defined piecewise
- ☐☐☐ use the general result $\mathrm{E}(g(X)) = \int f(x)g(x)\,\mathrm{d}x$ where $f(x)$ is the **probability density function** of the continuous random variable X and $g(X)$ is a function of X
- ☐☐☐ understand and use the relationship between the **probability density function** (PDF) and the **cumulative distribution function** (CDF), and use either to evaluate probabilities or percentiles
- ☐☐☐ use **cumulative distribution functions** (CDFs) of related variables in simple cases.

4.2 Inference using normal and t-distributions 利用正态分布与 t 分布的推断

- ☐☐☐ formulate hypotheses and apply a **hypothesis test** concerning the population mean using a small sample drawn from a normal population of unknown variance, using a **t-test**
- ☐☐☐ calculate a **pooled estimate** of a population variance from two samples
- ☐☐☐ formulate hypotheses concerning the difference of population means, and apply, as appropriate: - a **2-sample t-test** - a **paired sample t-test** - a test using a **normal distribution**
- ☐☐☐ determine a **confidence interval** for a population mean, based on a small sample from a normal population with unknown variance, using a **t-distribution**
- ☐☐☐ determine a **confidence interval** for a difference of population means, using a **t-distribution** or a **normal distribution**, as appropriate.

4.3 Chi-squared tests 卡方检验

- ☐☐☐ fit a theoretical distribution, as prescribed by a given hypothesis, to given data
- ☐☐☐ use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding **goodness of fit** analysis
- ☐☐☐ use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a **contingency table**.

4.4 Non-parametric tests 非参数检验

- ☐☐☐ understand the idea of a **non-parametric test** and appreciate situations in which such a test might be useful
- ☐☐☐ understand the basis of the **sign test**, the **Wilcoxon signed-rank test** and the **Wilcoxon rank-sum test**
- ☐☐☐ use a single-sample **sign test** and a single-sample **Wilcoxon signed-rank test** to test a hypothesis concerning a population **median**
- ☐☐☐ use a paired-sample **sign test**, a **Wilcoxon matched-pairs signed-rank test** and a **Wilcoxon rank-sum test**, as appropriate, to test for identity of populations.

4.5 Probability generating functions 概率母函数

- ☐☐☐ understand the concept of a **probability generating function** (PGF) and construct and use the PGF for given distributions
- ☐☐☐ use formulae for the mean and variance of a discrete random variable in terms of its PGF, and use these formulae to calculate the mean and variance of a given probability distribution
- ☐☐☐ use the result that the PGF of the sum of independent random variables is the product of the PGFs of those random variables.