

7(a)	$G_X(t) = a + 2at + bt^2$
	$G_X'(t) = 2a + 2bt$
	$E(X) = G_X'(1) = 2a + 2b$
7(b)	$G_X''(t) = 2b$
	$\text{Var}(X) = 2b + (2a + 2b) - (2a + 2b)^2$
	$2b + (2a + 2b) - (2a + 2b)^2 = 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$
	Alternative method for question 7(b)
	$E(X^2) = 0^2 \times a + 1^2 \times 2a + 2^2 \times b = 2a + 4b$
	$\text{Var}(X) = (2a + 4b) - (2a + 2b)^2$
	$(2a + 4b) - (2a + 2b)^2 = 2b + 2a + 2b - (2a + 2b)^2$ $= 2b + (2a + 2b)(1 - (2a + 2b))$ $= 2b + 2(a + b)(1 - 2a - 2b)$

7(c)	$G_Y(t) = (a + 2at + bt^2)^{10}$
	$G_Y'(t) = 20(a + bt)(a + 2at + bt^2)^9$
	$E(Y) = 20(a + b)(3a + b)^9$
	$3a + b = 1$, hence $E(Y) = 20(a + b) = 10E(X)$.
7(d)	Binomial
	$B\left(10, \frac{2}{3}\right)$

6(a)	$G_X'(t) = \frac{(3 - 2t)^2 + 4(3 - 2t)t}{(3 - 2t)^4} \left[= \frac{3 + 2t}{(3 - 2t)^3} \right]$
	$E(X) = G_X'(1) = 5$
	$G_X''(t) = \frac{2(3 - 2t)^3 + 6(3 + 2t)(3 - 2t)^2}{(3 - 2t)^6} \left[= \frac{24 + 8t}{(3 - 2t)^4} \right]$
	$\text{Var}(X) = G_X''(1) + G_X'(1) - (G_X'(1))^2 = 32 + 5 - 25$
	$[\text{Var}(X) =] 12$

6(b)	$G_Z(t) = \frac{t^3}{(3-2t)^4} = \frac{1}{81}t^3 \left(1 + (-4)\left(-\frac{2}{3}t\right) + \dots\right)$
	$P(Z=4) = \text{their coefficient of } t^4.$ OR $P(Z=3) = \text{their coefficient of } t^3.$
	$\left[P(Z=4)=\right] \frac{8}{243} \text{ and } \left[P(Z=3)=\right] \frac{1}{81}$
	$P(Z>4) = 1 - \left[\frac{8}{243}\right] - \left[\frac{1}{81}\right]$
	$P(Z>4) = \frac{232}{243}$

4(a)	$G_X(t) = \frac{2}{5}t + \frac{3}{5}t^2$
4(b)	$G_Y(t) = \frac{1}{625}t^4(2+3t)^4 \left[=t^4\left(\frac{2}{5}+\frac{3}{5}t\right)^4\right]$
4(c)	Attempt at coefficient of t^6 in <i>their</i> $G_Y(t)$.
	$P(Y=6) = \frac{216}{625}$

4(d)	$G_Y'(t) = \frac{4}{625}t^3(2+3t)^4 + \frac{12}{625}t^4(2+3t)^3$ OR $G_Y'(t) = \frac{1}{625}(64t^3 + 480t^4 + 1296t^5 + 1512t^6 + 648t^7)$
	$G_Y''(t) = \frac{12}{625}t^2(2+3t)^4 + \frac{96}{625}t^3(2+3t)^3 + \frac{108}{625}t^4(2+3t)^2$ OR $G_Y''(t) = \frac{1}{625}(192t^2 + 1920t^3 + 6480t^4 + 9072t^5 + 4536t^6)$
	$G_Y'(1) = \frac{32}{5} = 6.4, G_Y''(1) = \frac{888}{25} = 35.52$
	$\text{Var}(Y) = \frac{888}{25} + \frac{32}{5} - \left(\frac{32}{5}\right)^2$
	$\text{Var}(Y) = \frac{24}{25} \quad [= 0.96]$
	Alternative method for question 4(d)
	$E(X) = \frac{2}{5} + \frac{3}{5} \times 2 = \frac{8}{5}$ or use $G_X'(t) = \frac{2}{5} + \frac{6}{5}t$.
	$E(X^2) = \frac{2}{5} + \frac{3}{5} \times 4 = \frac{14}{5}$ or use $G_X''(t) = \frac{6}{5}$
	$\text{Var}(X) = \frac{14}{5} - \left(\frac{8}{5}\right)^2$ or $\text{Var}(X) = \frac{6}{5} + \frac{8}{5} - \left(\frac{8}{5}\right)^2$
	$\text{Var}(X) = \frac{6}{25}$
	$\text{Var}(Y) = 4\text{Var}(X) = \frac{24}{25}$

6(a)	$k = 1 - a$
6(b)	$(1 - at^2)^{-1} = 1 + at^2 + \dots$
	$P(Y > 2) = 1 - k(1 + a)$
	$P(Y > 2) = 1 - (1 - a)(1 + a) = a^2$
	Alternative method for question 6(b)
	$P(Y = 0) = G_Y(0) = k$ $\left[P(Y = 1) = G_Y'(0) = 0 \right]$
	$P(Y = 2) = \frac{1}{2}G_Y''(0) = \frac{1}{2}(2ak).$
	$P(Y > 2) = 1 - k - ak$
	$P(Y > 2) = 1 - (1 - a) - a(1 - a) = a^2$
6(c)	$G_Y'(t) = \frac{0.32t}{(1 - 0.2t^2)^2}$
	$E(Y) = G_Y'(1) = \frac{0.32}{(1 - 0.2)^2} = 0.5$

6(a)	Probabilities: $\frac{1}{15}, \frac{7}{15}, \frac{7}{15}$ for 0, 1, 2 heads.																			
	$G_X(t) = \frac{1}{15} + \frac{7}{15}t + \frac{7}{15}t^2$																			
	Probabilities: $\frac{8}{15}, \frac{7}{15}$ for 1, 2 colours.																			
	$G_Y(t) = \frac{8}{15}t + \frac{7}{15}t^2$																			
6(b)	X and Y are not independent.																			
6(c)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>1</td><td>2</td><td>1</td></tr><tr><td>$z = x + y$</td><td>1</td><td>3</td><td>3</td></tr><tr><td>$P(Z = z)$</td><td>$\frac{1}{15}$</td><td>$\frac{7}{15}$</td><td>$\frac{7}{15}$</td></tr></table>	x	0	1	2	y	1	2	1	$z = x + y$	1	3	3	$P(Z = z)$	$\frac{1}{15}$	$\frac{7}{15}$	$\frac{7}{15}$			
	x	0	1	2																
	y	1	2	1																
	$z = x + y$	1	3	3																
	$P(Z = z)$	$\frac{1}{15}$	$\frac{7}{15}$	$\frac{7}{15}$																
$G_Z(t) = \frac{1}{15}t + \frac{14}{15}t^3$																				
6(d)	$G_Z'(t) = \frac{1}{15} + \frac{42}{15}t^2$																			
	$G_Z'(1) = \frac{1}{15} + \frac{42}{15}$																			
	$\frac{43}{15} [= 2.87]$																			