7(a)	$\mathrm{G}_X(t) = a + 2at + bt^2$
	$\mathrm{G}_X'(t) = 2a + 2bt$ $\mathrm{E}(X) = \mathrm{G}_X'(1) = 2a + 2b$
7(b)	$\mathrm{G}_X''(t) = 2b$
	$\mathrm{Var}(X) = 2b + (2a+2b) - (2a+2b)^2$
	$2b + (2a+2b) - (2a+2b)^2 = 2b + (2a+2b)\left(1-(2a+2b)\right)$ $= 2b + 2(a+b)(1-2a-2b)$
	Alternative method for question 7(b)
	$\mathrm{E}\left(X^2\right) = 0^2 \times a + 1^2 \times 2a + 2^2 \times b = 2a + 4b$
	$\mathrm{Var}(X) = (2a+4b) - (2a+2b)^2$
	$(2a+4b) - (2a+2b)^2 = 2b + 2a + 2b - (2a+2b)^2$ $= 2b + (2a+2b)\left(1-(2a+2b)\right)$ $= 2b + 2(a+b)(1-2a-2b)$

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7(c)	$\mathrm{G}_Y(t)=\left(a+2at+bt^2\right)^{10}$
	$\mathrm{G}_Y^{'}(t)=20(a+bt)\left(a+2at+bt^2\right)^9$ $\mathrm{E}(Y)=20(a+b)(3a+b)^9$
	$3a+b=1$, hence $\mathrm{E}(Y)=20(a+b)=10\mathrm{E}(X)$.
7(d)	Binomial
	$\mathrm{B}\left(10,\dfrac{2}{3}\right)$

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6(a)	$\mathrm{G}_X^{'}(t)=\dfrac{(3-2t)^2+4(3-2t)t}{(3-2t)^4}\left[=\dfrac{3+2t}{(3-2t)^3}\right]$
	$\mathrm{E}(X)=\mathrm{G}_X^{'}(1)=5$
	$\mathrm{G}_X^{''}(t)=\dfrac{2(3-2t)^3+6(3+2t)(3-2t)^2}{(3-2t)^6}\left[=\dfrac{24+8t}{(3-2t)^4}\right]$
	$\mathrm{Var}(X)=\mathrm{G}_X^{''}(1)+\mathrm{G}_X^{'}(1)-\left(\mathrm{G}_X^{'}(1)\right)^2=32+5-25$
	$\left[\mathrm{Var}(X)=\right]12$

6(b)	$\mathrm{G}_Z(t)=\dfrac{t^3}{(3-2t)^4}=\frac{1}{81}t^3\left(1+(-4)\left(-\frac{2}{3}t\right)+\ldots\right)$
	$\mathrm{P}(Z=4)$ = *their* coefficient of t^4. OR $\mathrm{P}(Z=3)$ = *their* coefficient of t^3.
	$\left[\mathrm{P}(Z=4)=\right]\frac{8}{243}$ and $\left[\mathrm{P}(Z=3)=\right]\frac{1}{81}$
	$\mathrm{P}(Z>4)=1-{'}\frac{8}{243}{'}-{'}\frac{1}{81}{'}$
	$\mathrm{P}(Z>4)=\frac{232}{243}$

4(a)	$\mathrm{G}_X(t)=\frac{2}{5}t+\frac{3}{5}t^2$
4(b)	$\mathrm{G}_Y(t)=\frac{1}{625}t^4(2+3t)^4 \quad \left[=t^4\left(\frac{2}{5}+\frac{3}{5}t\right)^4\right]$
4(c)	Attempt at coefficient of t^6 in *their* $\mathrm{G}_Y(t)$.
	$\mathrm{P}(Y=6)=\frac{216}{625}$

4(d)	$\mathrm{G}_Y'(t)=\frac{4}{625}t^3(2+3t)^4+\frac{12}{625}t^4(2+3t)^3$ OR $\mathrm{G}_Y'(t)=\frac{1}{625}\left(64t^3+480t^4+1296t^5+1512t^6+648t^7\right)$
	$\mathrm{G}_Y''(t)=\frac{12}{625}t^2(2+3t)^4+\frac{96}{625}t^3(2+3t)^3+\frac{108}{625}t^4(2+3t)^2$ OR $\mathrm{G}_Y''(t)=\frac{1}{625}\left(192t^2+1920t^3+6480t^4+9072t^5+4536t^6\right)$
	$\mathrm{G}_Y'(1)=\frac{32}{5}=6.4,\ \mathrm{G}_Y''(1)=\frac{888}{25}=35.52$
	$\mathrm{Var}(Y)=\frac{888}{25}+\frac{32}{5}-\left(\frac{32}{5}\right)^2$
	$\mathrm{Var}(Y)=\frac{24}{25}\quad[=0.96]$
	Alternative method for question 4(d)
	$\mathrm{E}(X)=\frac{2}{5}+\frac{3}{5}\times2=\frac{8}{5}$ or use $\mathrm{G}_X'(t)=\frac{2}{5}+\frac{6}{5}t$.
	$\mathrm{E}\left(X^2\right)=\frac{2}{5}+\frac{3}{5}\times4=\frac{14}{5}$ or use $\mathrm{G}_X''(t)=\frac{6}{5}$
	$\mathrm{Var}(X)=\frac{14}{5}-\left(\frac{8}{5}\right)^2$ or $\mathrm{Var}(X)=\frac{6}{5}+\frac{8}{5}-\left(\frac{8}{5}\right)^2$
	$\mathrm{Var}(X)=\frac{6}{25}$
	$\mathrm{Var}(Y)=4\mathrm{Var}(X)=\frac{24}{25}$

6(a)	$k = 1 - a$
6(b)	$\left(1 - at^2\right)^{-1} = 1 + at^2 \left(+\ldots\right)$
	$\mathrm{P}\left(Y > 2\right) = 1 - k\left(1 + a\right)$
	$\mathrm{P}\left(Y > 2\right) = 1 - \left(1 - a\right)\left(1 + a\right) = a^2$
	Alternative method for question 6(b)
	$\mathrm{P}\left(Y = 0\right) = \mathrm{G}_Y\left(0\right) = k$ $\left[P(Y = 1) = \mathrm{G}'_Y\left(0\right) = 0\right]$ $\mathrm{P}\left(Y = 2\right) = \frac{1}{2}\mathrm{G}''_Y\left(0\right) = \frac{1}{2}(2ak).$
	$\mathrm{P}\left(Y > 2\right) = 1 - k - ak$
	$\mathrm{P}\left(Y > 2\right) = 1 - \left(1 - a\right) - a\left(1 - a\right) = a^2$
6(c)	$\mathrm{G}'_Y\left(t\right) = \dfrac{0.32t}{\left(1 - 0.2t^2\right)^2}$
	$\mathrm{E}\left(Y\right) = \mathrm{G}'_Y\left(1\right) = \dfrac{0.32}{\left(1 - 0.2\right)^2} = 0.5$

<table>
<tr><td rowspan="5">6(a)</td><td>Probabilities: $\frac{1}{15}$, $\frac{7}{15}$, $\frac{7}{15}$ for 0, 1, 2 heads.</td></tr>
<tr><td>$\mathrm{G}_X(t) = \frac{1}{15} + \frac{7}{15}t + \frac{7}{15}t^2$</td></tr>
<tr><td>Probabilities: $\frac{8}{15}$, $\frac{7}{15}$ for 1, 2 colours.</td></tr>
<tr><td>$\mathrm{G}_Y(t) = \frac{8}{15}t + \frac{7}{15}t^2$</td></tr>
<tr><td></td></tr>
<tr><td rowspan="2">6(b)</td><td>X and Y are not independent.</td></tr>
<tr><td></td></tr>
<tr><td rowspan="2">6(c)</td><td>
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>1</td><td>2</td><td>1</td></tr>
<tr><td>$z = x + y$</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td>$\mathrm{P}(Z = z)$</td><td>$\frac{1}{15}$</td><td>$\frac{7}{15}$</td><td>$\frac{7}{15}$</td></tr>
</table>
</td></tr>
<tr><td>$\mathrm{G}_Z(t) = \frac{1}{15}t + \frac{14}{15}t^3$</td></tr>
<tr><td rowspan="3">6(d)</td><td>$\mathrm{G}_Z'(t) = \frac{1}{15} + \frac{42}{15}t^2$
$\mathrm{G}_Z'(1) = \frac{1}{15} + \frac{42}{15}$</td></tr>
<tr><td>$\frac{43}{15}\ [= 2.87]$</td></tr>
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