

7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $P(X = r)$ as given in the following table.

r	0	1	2
$P(X = r)$	a	$2a$	b

(a) Write down the probability generating function of X , and use it to find an expression for $E(X)$ in terms of a and b . [2]

(b) Show that $\text{Var}(X) = 2b + 2(a + b)(1 - 2a - 2b)$. [3]

The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \dots + X_{10}$ where $X_1, X_2, X_3, \dots, X_{10}$ are ten independent observations of X .

(c) Using the probability generating function of Y , and your answer to part (a), show that $E(Y) = 10E(X)$. [3]

(d) For the case $b = 0$, define fully the distribution of Y . [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

③

- 6 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = \frac{t}{(3-2t)^2}.$$

- (a) Find $E(X)$ and $\text{Var}(X)$.

[5]

④

Additional page

$$G_Y(t) = \frac{t^2}{(3-2t)^2}.$$

If you use the following page to complete the answer to any question, the question number must be clearly shown.

- [5]

- (a) Write down the probability generating function $G_X(t)$ of X .

[1]

(b) Find the probability generating function $G_Y(t)$ of Y . Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

[2]

- (c) Use $G_Y(t)$ to find $P(Y = 6)$.

[2]

- (d) Find $\text{Var}(Y)$.

[5]

6 Y is a discrete random variable which takes the values $0, 2, 4, \dots$. The probability generating function of Y is given by

$$G_Y(t) = \frac{k}{1 - at^2}.$$

- (a) Find k in terms of a .

[1]

- (b) Show that $P(Y > 2) = a^2$.

[3]

It is now given that $a = 0.2$.

- (c) Find the value of $E(Y)$.

[2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

- 6 A bag contains 7 red balls and 3 blue balls. Kieran selects 2 balls at random, without replacement. The number of red balls selected by Kieran is denoted by X , and the number of different colours present in Kieran's selection is denoted by Y .

- (a) Find the probability generating functions, $G_X(t)$ of X and $G_Y(t)$ of Y . [4]

Additional page

(d) Use the probability generating function of Z to find $E(Z)$. [2]