

7

 r

O

1

2

$$P(X = r)$$
 q $2a$
$$b$$

- (a)

[illegible]

- (b)

1

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The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \dots + X_{10}$ where $X_1, X_2, X_3, \dots, X_{10}$ are ten independent observations of X .

- (c) Using the probability generating function of Y , and your answer to part (a), show that $E(Y) = 10E(X)$. [3]

[illegible]

- (d)** For the case $b = 0$, define fully the distribution of Y . [2]

This image shows a blank sheet of white paper designed for handwriting practice. It features ten evenly spaced horizontal dashed lines across its width. At the bottom center, there is a small circle containing the number '2'. The rest of the page is empty.

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.

6 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = \frac{t}{(3-2t)^2}.$$

(a) Find $E(X)$ and $\text{Var}(X)$.

[5]

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Additional page

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Name:

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- (a)** Write down the probability generating function $G_X(t)$ of X .

[1]

The random variable Y is the sum of four independent observations of X .

- (b) Find the probability generating function $G_Y(t)$ of Y . Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

[2]

.....

.....

.....

- (c)** Use $G_Y(t)$ to find $P(Y = 6)$.

[2]

[illegible]

(d)

[5]

8

6

$$G_Y(t) = \frac{k}{1 - at^2}.$$

(a) Find k in terms of a . [1]

(b) Show that $P(Y > 2) = a^2$. [3]

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Q

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It is now given that $a = 0.2$.

(c) Find the value of $E(Y)$.

[2]

Handwriting practice lines on page 10.

Additional page

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- 6

(a)

[illegible]

The random variable Z is the sum of the number of red balls and the number of different colours present in Kieran's selection. Kieran claims that the probability generating function of Z is equal to $G_Y(t) \times G_Y(t)$.

(b)

12

(c) Find the probability generating function of Z , expressing your answer as a polynomial in t . [4]

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(d) Use the probability generating function of Z to find $E(Z)$. [2]

[illegible]

Additional page

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