1(a)	$s^2 = \frac{1}{9}\left('71314' - \frac{'844'^2}{10}\right) \quad \left[= \frac{134}{15} = 8.933\right]$
	2.262
	$\frac{'844'}{10} \pm '2.262'\sqrt{\frac{'8.933'}{10}}$
	[82.3, 86.5]
	The distribution of estimates of angles is normal. OR The estimates are a random sample from some population. OR The estimates are independent. OR Underlying distribution is normal.
1(b)	Population unlikely to be normal as θ is close to right-angle / data is skewed. OR Estimates may not be independent, for example due to collusion. OR No indication that sample is random.

3	$\sum(x-\bar{x})^2 = 9\times 0.36 = 3.24$
	$'55.8' - \frac{'16.6'^2}{5}$
	$\frac{3.24+0.688}{10+5-2}(=0.302)$
	$s = 0.550$

6(a)	$H_0: \mu_B = \mu_A \quad H_1: \mu_B > \mu_A$
	Differences: 5, 0, –1, 6, 1, –2, 8, 7, 15
	$s_d^2 = \frac{1}{8}\left('405' - \frac{'39'^2}{9}\right) \quad [= 29.5]$
	$\dfrac{\frac{'39'}{9}}{\sqrt{\frac{'29.5'}{9}}} = 2.393$
	$'2.393' > 1.860$, reject H_0 / significant.
	Sufficient evidence to suggest new training programme results in reduced times. OR Sufficient evidence to support the coach's belief.
6(b)	The population of differences may not be normally distributed. The population of differences may not be symmetrical (and hence not normal).
6(c)	A Wilcoxon matched-**pairs** signed-rank test. OR A **paired**-sample sign test.

2(a)	$s^2 = \frac{1}{11}\left('476117' - \frac{'2390'^2}{12}\right) \quad \left[= \frac{326}{33} = 9.8788\right]$
	3.106
	$\frac{'2390'}{12} \pm '3.106'\sqrt{\frac{'9.8788'}{12}}$
	$[196,\ 202]$
2(b)	-1.796
	$\frac{'199.17' - k}{\sqrt{\frac{'9.8788'}{12}}} > '-1.796'$ or $\frac{\lvert '199.17' - k \rvert}{\sqrt{\frac{'9.8788'}{12}}} < '1.796'$
	200.8

5	$H_0: \mu_A = \mu_B \quad H_1: \mu_A \neq \mu_B$
	$'1331848' - \frac{'3264'^2}{8}\ [= 136]$
	$'964964' - \frac{'2406'^2}{6}\ [= 158]$
	$s_p^2 = \frac{'136' + '158'}{8+6-2} = 24.5$
	$t = \frac{\frac{'3264'}{8} - \frac{'2406'}{6}}{'s_p'\sqrt{\frac{1}{8}+\frac{1}{6}}} = 2.62$
	$'2.62' > 2.179$, reject H_0 / significant.
	Sufficient evidence to suggest that there is a difference in the mean tensile strengths of steel rods produced by the two machines.

1(a)	$\sum x = 723,\ \sum x^2 = 42261$
	$s^2 = \frac{1}{12}\left('42261' - \frac{'723'^2}{13}\right)\ \left[= \frac{2222}{13} = 170.9231\right]$
	1.782
	$\frac{'723'}{13} \pm '1.782'\sqrt{\frac{'170.9231'}{13}}$
	$[49.2, 62.1]$
1(b)	The population (of car speeds) is normally distributed. OR Underlying distribution is normal.

3(a)	$H_0: \mu_Y = \mu_X \quad H_1: \mu_Y > \mu_X$
	Differences: 3, –1, 6, 2, 3, 3, –1, –2, 3, 3
	$s_d^2 = \frac{1}{9}\left('91' - \frac{'19'^2}{10}\right) \quad [= 6.1]$
	$\dfrac{\frac{'19'}{10}}{\sqrt{\frac{'6.1'}{10}}} = 2.43$
	$'2.43' < 2.821$, accept H_0 / do not reject H_0 / not significant.
	Insufficient evidence to support the manufacturer's claim. OR Insufficient evidence to support that (on average) machine *Y* extracts more juice than machine *X*.

3(b)	Population of differences is normally distributed. OR Underlying distribution of differences is normal.
3(c)	The difference remains the same for pair 4. The set of differences has not changed. OR Redo the test: $2.43 < 2.821$
	The conclusion remains the same.

1	$\mathrm{H}_0 : \mu_x = \mu_y \qquad \mathrm{H}_1 : \mu_x \neq \mu_y$
	$s_x^2 = 103.218...$ $s_y^2 = 171.139...$
	$\dfrac{s_x^2}{120} + \dfrac{s_y^2}{80} \quad [= 2.999]$
	$[z =]\dfrac{\frac{10470}{120} - \frac{6560}{80}}{\sqrt{\frac{s_x^2}{120} + \frac{s_y^2}{80}}} = 3.03$
	'3.03' > 2.576, reject H_0 (significant). OR $\mathrm{P}(Z > \text{'3.03'}) = 0.00122 < 0.005$, reject H_0 (significant).
	There is sufficient evidence to suggest the **mean** spend of customers is different on weekends compared to weekdays.

4(a)	$\sum(x-\bar{x})^2 = 40104 - \frac{1}{10}\times 612^2$ $[=2649.6]$ $\sum(y-\bar{y})^2 = 27460 - \frac{1}{8}\times 444^2$ $[=2818]$
	$s_p^2 = \dfrac{2649.6+2818}{10+8-2}[=341.725]$
	$(61.2-55.5)\pm 2.120\sqrt{'s_p^2'\left(\frac{1}{10}+\frac{1}{8}\right)}[=5.7\pm 2.12\times 8.7686]$
	$[-12.9,\ 24.3]$
4(b)	Because zero lies inside the confidence interval.

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2	$s_x^2 = \frac{1}{39}\left(720 - \frac{168^2}{40}\right) = 0.36923$ $\left[=\frac{24}{65}\right]$ $s_y^2 = \frac{1}{59}\left(900 - \frac{228^2}{60}\right) = 0.56949$ $\left[=\frac{168}{295}\right]$
	$s^2 = \frac{'0.36923'}{40} + \frac{'0.56949'}{60}$
	$s^2 = 0.0187$ $\left[=\frac{359}{19175}\right]$
	$\mathrm{CI} = \left(\frac{168}{40} - \frac{228}{60}\right) \pm 1.645's'$
	$[0.175, 0.625]$

5(a)	$H_0 : \mu_B - \mu_A = k, \quad H_1 : \mu_B - \mu_A > k$
	Differences, d : 19, 17, 8, 16, 9, 23, 6, 5.
	$\sum d = 103, \quad \sum d^2 = 1641,$ $\bar{d} = 12.875, \quad s_d^2 = \frac{1}{7}\left(1641 - \frac{1}{8}\left(103^2\right)\right) = 44.982 \quad \left[= \frac{2519}{56}\right]$
	$t = \dfrac{12.875 - k}{\sqrt{\dfrac{s_d^2}{8}}}$
	$\dfrac{12.875 - k}{\sqrt{\dfrac{s_d^2}{8}}} \geqslant 1.895$
	$[0 \leqslant]k \leqslant 8.38$
5(b)	Population of differences are normally distributed.