

5(a)	$\frac{1}{16} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4 + \frac{1}{k} \left[2\sqrt{x} \right]_4^9 = 1$ $\frac{1}{24}(8-0) + \frac{2}{k}(3-2) = 1$ OR $\frac{1}{3} + \frac{2}{k} = 1$
	$\frac{2}{k} = \frac{2}{3}, k = 3$
5(b)	$\frac{1}{16} \int_0^4 \sqrt{x} \, dx + \frac{1}{3} \int_4^m x^{-\frac{1}{2}} \, dx = \frac{1}{2}$ OR $\frac{1}{3} \int_m^9 x^{-\frac{1}{2}} \, dx = 0.5$
	$\frac{1}{3} + \frac{2}{3}(\sqrt{m} - 2) = \frac{1}{2}$ OR $\frac{2}{3}(3 - \sqrt{m}) = \frac{1}{2}$
	$m = \frac{81}{16} \quad [= 5.0625]$

5(c)

$$F(x) = \begin{cases} \frac{1}{24}x^{\frac{3}{2}} & 0 \leq x < 4 \\ \frac{2}{3}x^{\frac{1}{2}} - 1 & 4 \leq x \leq 9 \end{cases}$$

$$G(y) = \begin{cases} \frac{1}{24}y^3 & 0 \leq y < 2 \\ \frac{2}{3}y - 1 & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Alternative method for question 5(c)

Using chain rule (or inverse function):

$$G(y) = F(y^2) \text{ so } g(y) = \frac{d}{dy} F(y^2) = 2yf(y^2).$$

$$g(y) = \begin{cases} 2y\left(\frac{1}{16}\sqrt{y^2}\right) & 0 \leq y < 2 \\ 2y\left(\frac{1}{3\sqrt{y^2}}\right) & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

4(a)	$a = -\frac{1}{5}$
	$b = \frac{7}{25}$
4(b)	$f(x) = \begin{cases} \frac{1}{5} & 1 \leq x < 4, \\ \frac{1}{25}x & 4 \leq x \leq 6, \\ 0 & \text{otherwise.} \end{cases}$
4(c)	$E(X^2) = \frac{1}{5} \int_1^4 x^2 dx + \frac{1}{25} \int_4^6 x^3 dx = \left[\frac{1}{15} x^3 \right]_1^4 + \left[\frac{1}{100} x^4 \right]_4^6$
	$\text{Var}(X) = \frac{73}{5} - \left(\frac{529}{150} \right)^2$
	2.16
4(d)	10th percentile: $\frac{1}{5}x - \frac{1}{5} = \frac{1}{10}$ 90th percentile: $\frac{1}{50}x^2 + \frac{7}{25} = \frac{9}{10}$
	10th percentile = 1.5
	90th percentile = $\sqrt{31}$

2(a)	$(F(x) =) \begin{cases} 0 & x < 0 \\ \frac{2}{9}x^2 + \frac{4}{9}x & 0 \leq x < 1 \\ \frac{1}{3}(x-2)^3 + 1 & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$
2(b)	$\frac{2}{9}m^2 + \frac{4}{9}m = \frac{1}{2}$ $m = \frac{1}{2}(-2 + \sqrt{13})$
2(c)	$(G(y) =) \begin{cases} 0 & y < 0 \\ \frac{2}{9}y^4 + \frac{4}{9}y^2 & 0 \leq y < 1 \\ \frac{1}{3}(y^2 - 2)^3 + 1 & 1 \leq y \leq \sqrt{2} \\ 1 & y > \sqrt{2} \end{cases}$

2(d)

$$G\left(\frac{1}{2}(-2 + \sqrt{13})\right) = 0.379$$

$0.379 < 0.5$ so the median of Y is greater than the median of X .

Alternative method for question 2(d)

$$m_y^2 = \frac{-2 + \sqrt{13}}{2}$$

$$m_y^2 = m_x < 1 \text{ and hence } m_y > m_x.$$

Alternative method for question 2(d)

$$\text{Solves } \frac{2}{9}y^4 + \frac{4}{9}y^2 = \frac{1}{2}.$$

$$m_y = 0.896 > 0.803 = m_x$$

2(a)	$5a + \frac{3}{2}a = 1$, leading to $a = \frac{2}{13}$.
	Attempt to solve 2 independent equations in b and c or 3 independent equations in a , b and c .
	$b = \frac{16}{39}$, $c = \frac{2}{39}$
2(b)	$[E(X) =] \int_0^5 \frac{2}{13}x \, dx + \int_5^8 x\left(\frac{16}{39} - \frac{2}{39}x\right) dx$
	$\left[\frac{1}{13}x^2\right]_0^5 + \left[\frac{8}{39}x^2 - \frac{2}{117}x^3\right]_5^8$
	$\frac{43}{13} = 3.31$
2(c)	$\frac{2}{13}m = 0.5$ OR $\int_0^m \frac{2}{13}dx = 0.5$
	$m = \frac{13}{4} = 3.25$
2(d)	$\frac{2}{13}x, \frac{16}{39}x - \frac{1}{39}x^2 (+k)$
	$k = -\frac{25}{39}$
	$G(y) = \begin{cases} 0 & y < 0 \\ \frac{2}{13}\sqrt{y} & 0 \leq y \leq 25 \\ \frac{16}{39}\sqrt{y} - \frac{1}{39}y - \frac{25}{39} & 25 < y \leq 64 \\ 1 & y > 64 \end{cases}$

3(a)	$\int_0^1 kx \, dx + \int_1^8 k(8-x) \, dx = 1 \quad \left[\left[\frac{1}{2} kx^2 \right]_0^1 + \left[8kx - \frac{1}{2} kx^2 \right]_1^8 = 1 \right]$ $\frac{1}{2}k + (64k - 32k) - \left(8k - \frac{1}{2}k \right) = 1$
	$25k = 1$, leading to $k = \frac{1}{25}$.
	Alternative method for question 3(a)
	$\frac{1}{2} \times 1 \times k + \frac{1}{2} \times 7 \times 7k = 1$
	$25k = 1$, leading to $k = \frac{1}{25}$.
3(b)	$\int_0^1 kx \, dx + \int_1^m k(8-x) \, dx = \frac{1}{2}$
	$m^2 - 16m + 39 = 0$ or
	$m = 3$
	Alternative method for question 3(b)
	$\frac{1}{2} \times (8-m) \times k(8-m) = \frac{1}{2}$
	$(8-m)^2 = 25$
	$m = 3$

3(c)

$$F(x) = \begin{cases} \frac{1}{50}x^2 & [0 \leq x < 1] \\ \frac{8}{25}x - \frac{1}{50}x^2 - \frac{7}{25} & [1 \leq x \leq 8] \end{cases}$$

[$F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 8$]

$$G(y) = \begin{cases} \frac{1}{50}y^6 & [0 \leq y < 1] \\ \frac{8}{25}y^3 - \frac{1}{50}y^6 - \frac{7}{25} & [1 \leq y \leq 2] \end{cases}$$

[$G(y) = 0$ for $x < 0$ and $G(y) = 1$ for $y > 2$]

$$g(y) = \begin{cases} \frac{3}{25}y^5 & 0 \leq y < 1 \\ \frac{24}{25}y^2 - \frac{3}{25}y^5 & 1 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Alternative method for question 3(c)

Let $p(y) = \sqrt[3]{y}$, then $q(y) = p^{-1}(y) = y^3$.

$$q'(y) = 3y^2.$$

$$g(y) = \begin{cases} \frac{1}{25}y^3(3y^2) & [0 \leq y < 1] \\ \frac{1}{25}(8 - y^3)(3y^2) & [1 \leq y \leq 2] \end{cases} \quad \text{or} \quad g(x) = \begin{cases} \frac{1}{25}x(3x^{\frac{2}{3}}) & [0 \leq x < 1] \\ \frac{1}{25}(8 - x)(3x^{\frac{2}{3}}) & [1 \leq x \leq 8] \end{cases}$$

($g(y) = 0$ otherwise) ($g(x) = 0$ otherwise)

$$g(y) = \begin{cases} \frac{3}{25}y^5 & 0 \leq y < 1 \\ \frac{24}{25}y^2 - \frac{3}{25}y^5 & 1 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$