

5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leq x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leq x \leq 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

(b) Find the median value of X . [3]

Further Mathematics: **9231 Mathematics - Further November 2025 Question Paper** 41

The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y .

[5]

2

4 A continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{1}{5}x + a & 1 \leq x < 4, \\ \frac{1}{50}x^2 + b & 4 \leq x \leq 6, \\ 1 & x > 6, \end{cases}$$

where a and b are constants.

(a) Find the value of a and the value of b . [2]

(b) Find the probability density function of X . [2]

(c) Given that $E(X) = \frac{529}{150}$, find $\text{Var}(X)$.

[3]

(d) Find the 10th and 90th percentiles of X .

[3]

2

$$f(x) = \begin{cases} \frac{4}{9}(x+1) & 0 \leq x \leq 1, \\ (x-2)^2 & 1 < x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a)

[3]

[illegible]

(b)

[2]

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The random variable Y is defined by $Y = \sqrt{X}$.

- (c)** Find the cumulative distribution function of Y . [3]

[illegible]

- (d) Determine whether the median of Y is greater than, or less than, the median of X . [2]

[illegible]

(b)

[3]

[illegible]

(c)

[2]

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The random variable Y is defined by $Y = X^2$.

(d)

[4]

[illegible]

3

A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx & 0 \leq x < 1, \\ k(8-x) & 1 \leq x \leq 8, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{25}$.

[2]

(b) Find the median value of X .

[3]

Further Mathematics: 9231 Mathematics - Further June 2025 Question Paper 43

The random variable Y is defined by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y .

[5]

10