

1

Let  $v_A$  and  $v_B$  be the speeds of  $A$  and  $B$  respectively after the collision.

$$4mv_A + mv_B = 4mu$$

$$-v_A + v_B = eu$$

$$v_A = \frac{3}{4}u, \quad v_B = u$$

$$e = \frac{1}{4}$$

### Alternative method for question 1

Let  $v_A$  and  $v_B$  be the speeds of  $A$  and  $B$  respectively after the collision.

$$4mv_A + mv_B = 4mu$$

$$-v_A + v_B = eu$$

$$v_A = \frac{1}{5}u(4 - e), \quad v_B = \frac{4}{5}u(1 + e)$$

$$\frac{4m\left(\frac{1}{5}u(4 - e)\right)}{m\left(\frac{4}{5}u(1 + e)\right)} = 3, \quad e = \frac{1}{4}$$

|      |                                                                 |
|------|-----------------------------------------------------------------|
| 5(a) | $u \sin \alpha = v \sin(2\alpha)$                               |
|      | $v = \frac{5}{8}u$                                              |
|      |                                                                 |
| 5(b) | $m \cos \alpha = m v \cos(2\alpha) + k m w$                     |
|      | $e \cos \alpha = w - v \cos(2\alpha)$                           |
|      | $\frac{5}{8k} = \frac{4}{5}e + \frac{5}{8} \times \frac{7}{25}$ |
|      | $e = \frac{25}{32k} - \frac{7}{32}$                             |
|      |                                                                 |
| 5(c) | $0 \leq \frac{25}{32k} - \frac{7}{32} \leq 1$                   |
|      | $7k \leq 25 \leq 39k$                                           |
|      | $\frac{25}{39} \leq k \leq \frac{25}{7}$                        |
|      |                                                                 |

7

Perpendicular to the wall:  $eu \sin \alpha = v \sin \theta$ 

$$P_1 P_2 = uT$$

$$P_2 P_3 = vkT$$

$$\sin \alpha = \frac{d}{P_1 P_2} \quad \sin \theta = \frac{d}{P_2 P_3}$$

$$\sin \alpha = \frac{d}{uT} \quad \sin \theta = \frac{d}{vkT}$$

$$\frac{eud}{uT} = \frac{vd}{vkT}$$

$$e = \frac{1}{k}$$

|      |                                                                                                                                                                                                                                                            |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(a) | $\frac{9}{2} \times \frac{1}{2} \times 2mv_A^2 = \frac{1}{2}mv_B^2 \quad [3v_A = v_B]$                                                                                                                                                                     |
|      | $2mu = 2mv_A + mv_B$                                                                                                                                                                                                                                       |
|      | $[v_A = \frac{2}{5}u] \quad v_B = \frac{6}{5}u$                                                                                                                                                                                                            |
|      |                                                                                                                                                                                                                                                            |
| 6(b) | Let $w$ denote the speed after the second collision.<br>Perpendicular to wall: $v_B \cos \alpha = w \cos \beta$                                                                                                                                            |
|      | Parallel to wall: $e v_B \sin \alpha = w \sin \beta$                                                                                                                                                                                                       |
|      | $e^2 v_B^2 \sin^2 \alpha = w^2 \sin^2 \beta \quad \left[ \frac{576}{625} \sin^2 \alpha = \frac{144}{125} \sin^2 \beta \right]$<br>$v_B^2 \cos^2 \alpha = w^2 \cos^2 \beta \quad \left[ \frac{36}{25} \cos^2 \alpha = \frac{144}{125} \cos^2 \beta \right]$ |
|      | $\frac{576}{625} \sin^2 \alpha + \frac{36}{25} (1 - \sin^2 \alpha) = \frac{144}{125}$                                                                                                                                                                      |
|      | $[\sin^2 \alpha = \frac{5}{9}] \quad \sin \alpha = \frac{1}{3} \sqrt{5}$                                                                                                                                                                                   |
|      | $\sin(\alpha + \beta) = 1$                                                                                                                                                                                                                                 |
|      |                                                                                                                                                                                                                                                            |

|      |                                                                                                                                                                      |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(a) | Let $v$ and $w$ denote the horizontal components of the velocities for $A$ and $B$ respectively after the collision.<br>$2mu \cos \theta - mu \cos \theta = mw + mv$ |
|      | $w - v = e(3u \cos \theta)$                                                                                                                                          |
|      | $w = \frac{1}{2}(3eu \cos \theta + u \cos \theta) \quad \left[ = \frac{1}{2}(3e + 1)u \cos \theta \right]$                                                           |
|      | $\frac{\frac{1}{2}(3e + 1)u \cos \theta}{u \sin \theta} = \tan \theta$                                                                                               |
|      | $\tan^2 \theta = \frac{1}{2}(3e + 1)$                                                                                                                                |
|      | $\tan \theta = \sqrt{\frac{1}{2}(3e + 1)}$                                                                                                                           |
|      |                                                                                                                                                                      |
| 6(b) | $w = 3eu \cos \theta = u \cos \theta$<br>$e = \frac{1}{3}$                                                                                                           |
|      | $\tan \theta = 1, \quad \theta = 45^\circ$                                                                                                                           |
|      | <b>Alternative method for question 6(b)</b>                                                                                                                          |
|      | $v = 0, \quad w = u \cos \theta \quad \text{and} \quad \frac{w}{u \sin \theta} = \tan \theta$                                                                        |
|      | $u \cos \theta = u \sin \theta \tan \theta, \quad \tan^2 \theta = 1, \quad \theta = 45^\circ$                                                                        |
|      |                                                                                                                                                                      |