

3(a)	$\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+ A]$
	$t = 0, v = 2 : \quad A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$
	$v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+ B]$
	$x = 0, t = 0 : \quad B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \quad \left[= \ln\left(2\cos^2\left(t - \frac{1}{4}\pi\right)\right) \right]$
3(b)	$2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$
	$t = \frac{1}{2}\pi$

7(a)	$-mgkv - \mu mg = m \frac{dv}{dt}$
	$t = -\frac{1}{g} \int \frac{1}{kv + \mu} dv$ $t = -\frac{1}{gk} \ln(kv + \mu) [+ C]$
	$\text{When } v = u, t = 0: \quad C = \frac{1}{gk} \ln(kU + \mu)$
	$t = -\frac{1}{gk} \ln(kv + \mu) + \frac{1}{gk} \ln(kU + \mu)$ $t = \frac{1}{gk} \ln\left(\frac{kU + \mu}{kv + \mu}\right)$

7(b)	$-\frac{2}{5}v - 2 = v \frac{dv}{dx}$ $x = -\frac{5}{2} \int \frac{v}{v+5} dv = -\frac{5}{2} \int \left(1 - \frac{5}{v+5}\right) dv$
	$x = -\frac{5}{2}v + \frac{25}{2} \ln(v+5) + C$
	When $x=0$, $v=10$: $C = 25 - \frac{25}{2} \ln(15)$
	When $v=0$, $x = 25 - \frac{25}{2} \ln(15) + \frac{25}{2} \ln(5) = 11.3$ [m]
	Alternative method for question 7(b)
	From part (a), $t = \frac{5}{2} \ln\left(\frac{15}{v+5}\right)$, $v = 15e^{-\frac{2}{5}t} - 5$
	$x = -\frac{75}{2}e^{-\frac{2}{5}t} - 5t + B$
	When $t=0$, $x=0$, $B = \frac{75}{2}$, $x = -\frac{75}{2}e^{-\frac{2}{5}t} - 5t + \frac{75}{2}$
	When $v=0$, $t = 2.5 \ln 3$, $x = 25 - \frac{25}{2} \ln 3 = 11.3$ [m]

7(b)	Alternative method for question 7(b)
	From part (a), $t = \frac{5}{2} \ln\left(\frac{15}{v+5}\right)$, $v = 15e^{-\frac{2}{5}t} - 5$
	When $v=0$, $t = 2.5 \ln 3$ so $x = \int_0^{2.5 \ln 3} (15e^{-\frac{2}{5}t} - 5) dt$
	$x = \left[-\frac{75}{2}e^{-\frac{2}{5}t} - 5t\right]_0^{2.5 \ln 3}$
	$x = -\frac{75}{2} \times \frac{1}{3} - 5 \times 12.5 \times \ln 3 - \left(-\frac{75}{2}\right) = 25 - \frac{25}{2} \ln 3 = 11.3$ [m]
7(c)	$\text{Average speed} = \frac{11.3}{\frac{1}{10 \times 0.04} \ln\left(\frac{0.04 \times 10 + 0.2}{0.2}\right)} = \frac{11.3}{2.5 \ln 3}$
	4.10 [ms ⁻¹]

5(a)	$mv \frac{dv}{dx} = -0.025v^2 m - 10m$
	$\int \frac{v}{10 + 0.025v^2} dv = - \int dx$ $-0.025x[+c] = \frac{1}{2} \ln(v^2 + 400)$
	$c = \frac{1}{2} \ln 800$
	$v = 20\sqrt{2e^{-\frac{1}{20}x} - 1}$
5(b)	$m \frac{dv}{dt} = -0.025v^2 m - 10m$ $- \int dt = \int \frac{1}{10 + 0.025v^2} dv$ $-t[+k] = 2 \tan^{-1} \frac{v}{20}$
	$k = 2 \tan^{-1} \frac{20}{20} = \frac{1}{2} \pi$
	$v = 20 \tan\left(\frac{1}{4} \pi - \frac{1}{2} t\right)$

1(a)	$x^3 + 4x = 8v \frac{dv}{dx}$
	$\int (x^3 + 4x) dx = \int 8v dv$ $4v^2 = \frac{1}{4}x^4 + 2x^2 + c$
	$v^2 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 \quad \left[= \left(\frac{1}{4}x^2 + 1\right)^2 \right]$ $v = \frac{1}{4}x^2 + 1$
1(b)	$\frac{dx}{dt} = \frac{1}{4}x^2 + 1 \text{ leading to } \int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int dt .$ $t = 4 \int \frac{1}{x^2 + 4} dx = 2 \tan^{-1}\left(\frac{1}{2}x\right)[+C]$
	$C = 0 \text{ so } x = 2 \tan\left(\frac{1}{2}t\right)$

3(a)	$-mg - mkv^2 = mv \frac{dv}{dx}$
	$\int dx = - \int \frac{v}{g + kv^2} dv$ leading to $x = -\frac{1}{2k} \ln(g + kv^2) [+c]$.
	$v = U$ when $x = 0$, so $c = \frac{1}{2k} \ln(g + kU^2)$.
	$x = \frac{1}{2k} \ln\left(\frac{g + kU^2}{g + kv^2}\right)$ (metres)
3(b)	$10 + \frac{1}{40}v^2 = -\frac{dv}{dt}$
	$\int dt = - \int \frac{40}{400 + v^2} dv$
	$t = -\frac{40}{20} \tan^{-1}\left(\frac{1}{20}v\right) (+C)$
	When $t = 0, v = 20$, so $C = \frac{1}{2}\pi$ $t = -2 \tan^{-1}(0) + \frac{1}{2}\pi$
	$t = 1.57$ (seconds)