

|      |                                                                                                                                                                            |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3(a) | $\frac{dv}{dt} = -\frac{1}{2}(v^2 + 4)$ $\frac{1}{2} \tan^{-1}\left(\frac{1}{2}v\right) = -\frac{1}{2}t [+A]$                                                              |
|      | $t = 0, v = 2 : A = \frac{1}{2} \tan^{-1}\left(\frac{2}{2}\right) = \frac{1}{8}\pi$                                                                                        |
|      | $v = \frac{dx}{dt} = 2 \tan\left(\frac{1}{4}\pi - t\right)$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) [+B]$                                               |
|      | $x = 0, t = 0 : B = \ln 2$ $x = 2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 \quad \left[= \ln\left(2\cos^2\left(t - \frac{1}{4}\pi\right)\right)\right]$ |
|      |                                                                                                                                                                            |
| 3(b) | $2 \ln\left(\cos\left(t - \frac{1}{4}\pi\right)\right) + \ln 2 = 0$                                                                                                        |
|      | $t = \frac{1}{2}\pi$                                                                                                                                                       |
|      |                                                                                                                                                                            |

7(a)

$$-mgkv - \mu mg = m \frac{dv}{dt}$$

$$t = -\frac{1}{g} \int \frac{1}{kv + \mu} dv$$

$$t = -\frac{1}{gk} \ln(kv + \mu) [+C]$$

When  $v = u$ ,  $t = 0$ :  $C = \frac{1}{gk} \ln(kU + \mu)$

$$t = -\frac{1}{gk} \ln(kv + \mu) + \frac{1}{gk} \ln(kU + \mu)$$

$$t = \frac{1}{gk} \ln\left(\frac{kU + \mu}{kv + \mu}\right)$$

7(b)

$$-\frac{2}{5}v - 2 = v \frac{dv}{dx}$$

$$x = -\frac{5}{2} \int \frac{v}{v+5} dv = -\frac{5}{2} \int \left(1 - \frac{5}{v+5}\right) dv$$

$$x = -\frac{5}{2}v + \frac{25}{2} \ln(v+5) [+C]$$

When  $x=0$ ,  $v=10$ :  $C = 25 - \frac{25}{2} \ln(15)$

When  $v=0$ ,  $x = 25 - \frac{25}{2} \ln(15) + \frac{25}{2} \ln(5) = 11.3$  [m]

**Alternative method for question 7(b)**

From part (a),  $t = \frac{5}{2} \ln\left(\frac{15}{v+5}\right)$ ,  $v = 15e^{-\frac{2}{5}t} - 5$

$$x = -\frac{75}{2} e^{-\frac{2}{5}t} - 5t [+B]$$

When  $t=0$ ,  $x=0$ ,  $B = \frac{75}{2}$ ,  $x = -\frac{75}{2} e^{-\frac{2}{5}t} - 5t + \frac{75}{2}$

When  $v=0$ ,  $t = 2.5 \ln 3$ ,  $x = 25 - \frac{25}{2} \ln 3 = 11.3$  [m]

|      |                                                                                                                                               |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b) | <b>Alternative method for question 7(b)</b>                                                                                                   |
|      | From part (a), $t = \frac{5}{2} \ln\left(\frac{15}{v+5}\right)$ , $v = 15e^{-\frac{2}{5}t} - 5$                                               |
|      | When $v = 0$ , $t = 2.5 \ln 3$ so $x = \int_0^{2.5 \ln 3} (15e^{-\frac{2}{5}t} - 5) dt$                                                       |
|      | $x = \left[-\frac{75}{2}e^{-\frac{2}{5}t} - 5t\right]_0^{2.5 \ln 3}$                                                                          |
|      | $x = -\frac{75}{2} \times \frac{1}{3} - 5 \times 12.5 \times \ln 3 - \left(-\frac{75}{2}\right) = 25 - \frac{25}{2} \ln 3 = 11.3 \text{ [m]}$ |
|      |                                                                                                                                               |
| 7(c) | Average speed = $\frac{11.3}{\frac{1}{10 \times 0.04} \ln\left(\frac{0.04 \times 10 + 0.2}{0.2}\right)} = \frac{11.3}{2.5 \ln 3}$             |
|      | 4.10 [ms <sup>-1</sup> ]                                                                                                                      |
|      |                                                                                                                                               |

|      |                                                                                                                              |
|------|------------------------------------------------------------------------------------------------------------------------------|
| 5(a) | $mv \frac{dv}{dx} = -0.025v^2 m - 10m$                                                                                       |
|      | $\int \frac{v}{10 + 0.025v^2} dv = -\int dx$<br>$-0.025x[+c] = \frac{1}{2} \ln(v^2 + 400)$                                   |
|      | $c = \frac{1}{2} \ln 800$                                                                                                    |
|      | $v = 20\sqrt{2e^{-\frac{1}{20}x} - 1}$                                                                                       |
|      |                                                                                                                              |
| 5(b) | $m \frac{dv}{dt} = -0.025v^2 m - 10m$<br>$-\int dt = \int \frac{1}{10 + 0.025v^2} dv$<br>$-t[+k] = 2 \tan^{-1} \frac{v}{20}$ |
|      | $k = 2 \tan^{-1} \frac{20}{20} = \frac{1}{2} \pi$                                                                            |
|      | $v = 20 \tan\left(\frac{1}{4} \pi - \frac{1}{2} t\right)$                                                                    |
|      |                                                                                                                              |

|      |                                                                                                                                                                                            |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(a) | $x^3 + 4x = 8v \frac{dv}{dx}$                                                                                                                                                              |
|      | $\int (x^3 + 4x) dx = \int 8v dv$ $4v^2 = \frac{1}{4}x^4 + 2x^2 + c$                                                                                                                       |
|      | $v^2 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 \quad \left[ = \left( \frac{1}{4}x^2 + 1 \right)^2 \right]$ $v = \frac{1}{4}x^2 + 1$                                                           |
|      |                                                                                                                                                                                            |
| 1(b) | $\frac{dx}{dt} = \frac{1}{4}x^2 + 1 \text{ leading to } \int \frac{1}{\frac{1}{4}x^2 + 1} dx = \int dt .$ $t = 4 \int \frac{1}{x^2 + 4} dx = 2 \tan^{-1} \left( \frac{1}{2}x \right) [+C]$ |
|      | $C = 0 \text{ so } x = 2 \tan \left( \frac{1}{2}t \right)$                                                                                                                                 |
|      |                                                                                                                                                                                            |

|      |                                                                                                    |
|------|----------------------------------------------------------------------------------------------------|
| 3(a) | $-mg - mkv^2 = mv \frac{dv}{dx}$                                                                   |
|      | $\int dx = - \int \frac{v}{g + kv^2} dv \text{ leading to } x = -\frac{1}{2k} \ln(g + kv^2) [+c].$ |
|      | $v = U \text{ when } x = 0, \text{ so } c = \frac{1}{2k} \ln(g + kU^2).$                           |
|      | $x = \frac{1}{2k} \ln\left(\frac{g + kU^2}{g + kv^2}\right) \text{ (metres)}$                      |
|      |                                                                                                    |
| 3(b) | $10 + \frac{1}{40}v^2 = -\frac{dv}{dt}$                                                            |
|      | $\int dt = - \int \frac{40}{400 + v^2} dv$                                                         |
|      | $t = -\frac{40}{20} \tan^{-1}\left(\frac{1}{20}v\right) (+C)$                                      |
|      | When $t = 0, v = 20$ , so $C = \frac{1}{2}\pi$<br>$t = -2 \tan^{-1}(0) + \frac{1}{2}\pi$           |
|      | $t = 1.57 \text{ (seconds)}$                                                                       |
|      |                                                                                                    |