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In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the length of the string when speed of P is $\sqrt{\frac{7}{5}ag}$.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mg(x-a)^2}{2a} = mg(e+a-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 60ax + 27a^2 = 0$$

$$x = \frac{9}{5}a \text{ only.}$$

Alternative method for question 4

In equilibrium, $\frac{5mge}{a} = 5mg$, $e = a$.

Let x be the extension when P is released.

Energy equation in subsequent motion:

$$\frac{5mga^2}{2a} - \frac{5mgx^2}{2a} = mg(e-x) + \frac{1}{2}m\left(\frac{7}{5}ag\right)$$

$$25x^2 - 10ax - 8a^2 = 0$$

$$(5x+2a)(5x-4a)=0, \quad x = \frac{4}{5}a \text{ only.}$$

Required length of string = $\frac{9}{5}a$.

2(a)

$$\text{EPE} = \frac{2mg}{2a} \left(\frac{1}{3}a\right)^2$$

$$\text{Work done by friction} = \mu mg \left(\frac{1}{3}a\right)$$

$$\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \mu mg \left(\frac{1}{3}a\right)$$

$$\mu = \frac{1}{3}$$

2(b)

Let x be the extension in the string when P comes to rest.

$$\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \frac{2mg}{2a} x^2 + \frac{1}{2}mg \left(\frac{1}{3}a - x\right)$$

$$\left[18x^2 - 9ax + a^2 = 0, \quad (6x-a)(3x-a)=0\right] \quad x = \frac{1}{6}a \text{ only.}$$

Alternative method for question 2(b)

Let x be the distance moved by P before coming to rest.

$$\frac{2mg}{2a} \left(\frac{1}{3}a\right)^2 = \frac{2mg}{2a} \left(\frac{1}{3}a - x\right)^2 + \frac{1}{2}mgx$$

$$6x^2 - ax = 0, \quad x = \frac{1}{6}a$$

$$\text{Extension} = \frac{1}{3}a - x = \frac{1}{6}a$$

6(a)	N2L: $T_1 = T_2 + kmg$
	$T_1 = \frac{5mg(22a - BP - 4a)}{4a}$
	$T_2 = \frac{6mg(BP - 8a)}{8a}$
	$\frac{5mg(22a - BP - 4a)}{4a} = \frac{6mg(BP - 8a)}{8a} + kmg$
	$8BP = 114a - 4ak$
	$BP = \frac{57a - 2ak}{4}$
	Alternative method for question 6(a)
	N2L: $T_1 = T_2 + kmg$
	$T_1 = \frac{5mgx}{4a}$
	$T_2 = \frac{6mg(18a - x - 8a)}{8a} \left[= \frac{6mg(10a - x)}{8a} \right]$
	$\frac{5mgx}{4a} = \frac{6mg(10a - x)}{8a} + kmg$
	$x = \frac{15a + 2ak}{4}$
	$BP = 18a - x = 18a - \frac{15a + 2ak}{4} = \frac{57a - 2ak}{4}$

6(a)	Alternative method for question 6(a)
	N2L: $T_1 = T_2 + kmg$
	$T_1 = \frac{5mg(x - 4a)}{4a}$
	$T_2 = \frac{6mg(22a - x - 8a)}{8a} \left[= \frac{6mg(14a - x)}{8a} \right]$
	$\frac{5mg(x - 4a)}{4a} = \frac{6mg(14a - x)}{8a} + kmg$
	$x = \frac{31a + 2ak}{4}, \quad BP = 22a - x = 22a - \frac{31a + 2ak}{4} = \frac{57a - 2ak}{4}$
	Alternative method for question 6(a)
	N2L: $T_1 = T_2 + kmg$
	$T_1 = \frac{5mg(10a - x)}{4a}$
	$T_2 = \frac{6mgx}{8a}$
	$\frac{5mg(10a - x)}{4a} = \frac{6mgx}{8a} + kmg$
	$x = \frac{25a - 2ak}{4}, \quad BP = 8a + x = 8a + \frac{25a - 2ak}{4} = \frac{57a - 2ak}{4}$

6(a)	Alternative method for question 6(a)
	N2L: $T_1 = T_2 + kmg$
	$T_1 = \frac{5mgx_A}{4a}$ or $T_2 = \frac{6mgx_B}{8a}$
	$T_1 = \frac{5mgx_A}{4a}$, $T_2 = \frac{6mgx_B}{8a}$, $x_A + x_B = 10a$
	$\frac{5mgx_A}{4a} = \frac{6mgx_B}{8a} + kmg$, $5x_A - 3x_B = 4k$ $x_A = \frac{15a + 2ak}{4}$ OR $x_B = \frac{25a - 2ak}{4}$ Either $BP = 18a - x_A = 18a - \frac{15a + 2ak}{4} = \frac{57a - 2ak}{4}$ OR $BP = 8a + x_A = 8a + \frac{25a - 2ak}{4} = \frac{57a - 2ak}{4}$
6(b)	$\frac{6mg(18a - 8a)^2}{2 \times 8a} + 18kmg a = \frac{5mg(14a - 4a)^2}{2 \times 4a} + 8kmg a$
	$k = \frac{5}{2}$

5	At v_{max} , $T = 2g \sin \alpha$ $\left[= \frac{14}{25}g \right]$
	$\frac{14x}{0.5} = \frac{14g}{25}$ leading to $x = \frac{1}{5}$.
	EPE at top: $\frac{14 \times 0.3^2}{2 \times 0.5} [= 1.26]$
	EPE at bottom: $\frac{14 \times ("their x")^2}{2 \times 0.5} [= 0.56]$
	$2g \sin \alpha (0.8 + 0.5 + "their x") + \frac{14 \times 0.3^2}{2 \times 0.5} = \frac{14 \times ("their x")^2}{2 \times 0.5} + \frac{1}{2} \times 2 \times v^2$
	$v^2 = 0.91g = 9.1$ leading to $v = 3.02 \text{ m s}^{-1}$.

2(a)	$\frac{mg}{2a}(x-a)^2 = mgx \sin 30 - \frac{11}{30}mgx$
	$15x^2 - 34ax + 15a^2 = 0$ $(3x-5a)(5x-3a) = 0$
	$x = \frac{5}{3}a$
2(b)	$T = \frac{2}{3}mg$
	$F = T - mg \sin 30 \quad \left[= \frac{2}{3}mg - \frac{1}{2}mg \right]$
	$\frac{1}{6}mg$